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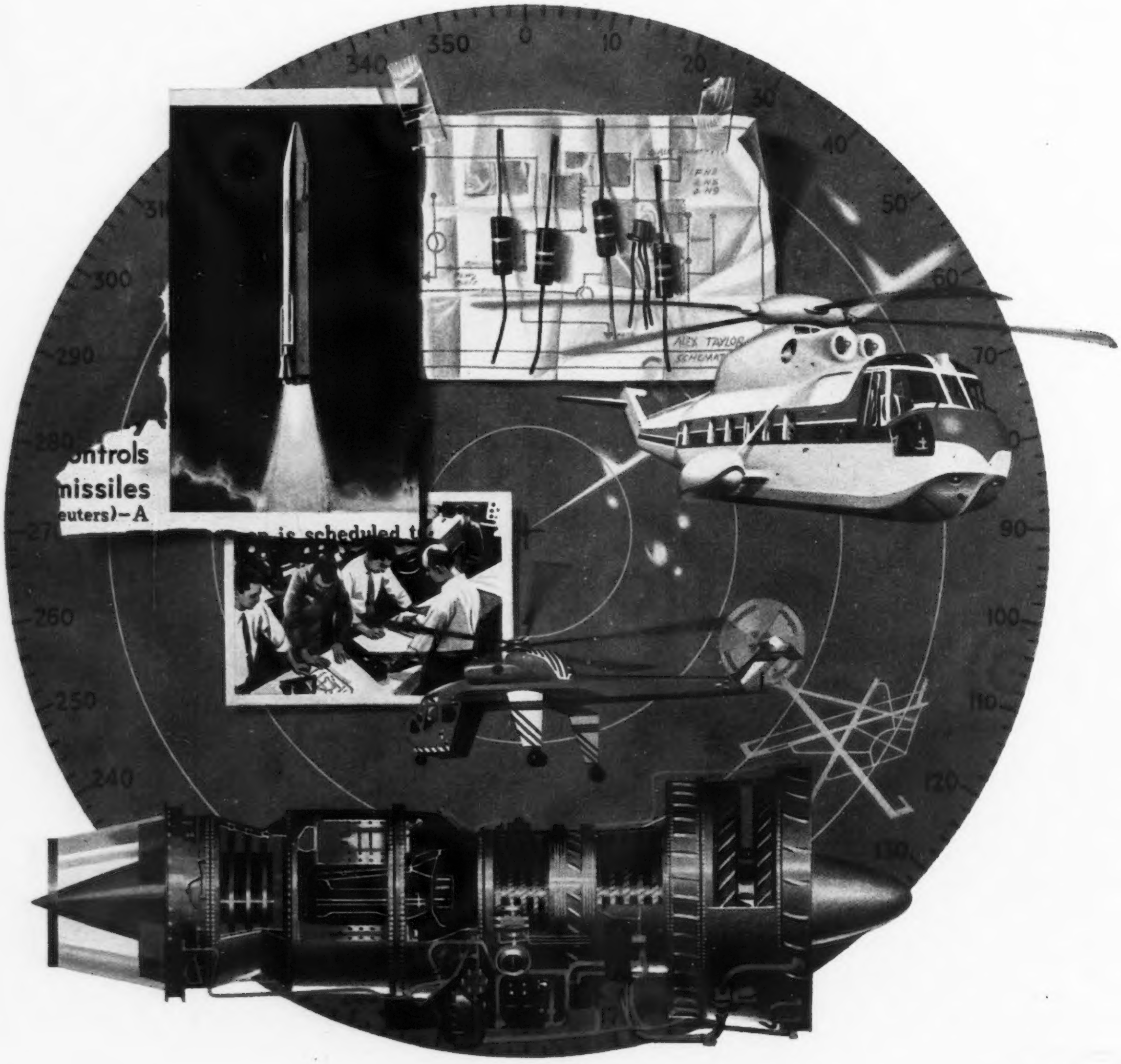

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CANADIAN AERONAUTICAL INSTITUTE

JOURNAL

NEW HORIZONS FOR CANADA'S AIR POLICY†

by G. W. G. McConachie*

Canadian Pacific Air Lines, Limited

Mr. G. W. G. McConachie

I WELCOME this privilege of addressing the Mid-season Meeting Dinner of the Canadian Aeronautical Institute, because it provides me with a long awaited opportunity to express my high regard for the Canadian aeronautical engineering talent and achievement which your organization represents.

As you know, I have been directly involved with most phases of Canadian aviation for more than a quarter of a century and thus my tribute is no mere polite expression, but represents an appraisal derived from my own experience and observation in the air industry.

You and your associates in the aeronautical sciences have proved that Canadian talent in original design as well as manufacturing is comparable with any in the world. It was this talent which upgraded Canada from the humble role of industrial satellite, merely copying the designs of others, to a position of prestige in original research, design and development as well as production.

Our great tragedy, of course, has been the limited scope and opportunity for the exercise of this talent in Canada, with the resulting drain of this valuable human resource to other countries. It is obvious that we must exercise the greatest enterprise and ingenuity in discovering and creating opportunities for our air industry, and in this I refer to both the manufacturer of aviation products and to their employment in air transportation.

As far as the manufacturing industry is concerned, it may well be that the requirement is for highly specialized activities, rather than attempts to compete with the big nations in the financially precarious and somewhat limited market for the big airliners. We are all familiar with the classic example of such Canadian designed aircraft as the Beaver, the Otter and the Caribou, all of which have been notably successful in capturing world markets.

It is no secret that I have always been and I am more than ever today a strong believer in a "buy

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†Dinner address read at the Mid-season Meeting of the C.A.I. in Winnipeg on the 27th February, 1961.
*President

Canadian" policy for Canadians, and I believe this should apply to Canadian airlines as well as to other individuals in this country.

It should not be necessary to emphasize this policy because it is so patently to the advantage of all of us to buy Canadian products whenever it is practical to do so. This does not mean, of course, that individuals or companies should be expected to buy Canadian or to fly Canadian unless the product is available at the time required, is offered at a reasonably competitive price and is of comparable quality to the foreign product.

Speaking in Hamilton early in December, I urged Canadians to buy Canadian and fly Canadian. It seems obvious to me that for self interest alone every Canadian businessman should fly Canadian because when he does he is protecting Canadian jobs and thus protects his own business and prosperity I emphasized that the price of the Canadian air transportation product is the same as the foreign product and that the Canadian quality is as high as any in the world.

I was puzzled to read in the current issue of *Aircraft* magazine a letter from a reader and an editorial which claimed that Canadian airlines, including CPAL, were not in fact following a "buy Canadian" policy. This conclusion was reached apparently from the fact that "neither of the major Canadian airlines has bought Canadian equipment for over a decade"

I have great respect for the Canadian aviation magazines, as I believe they have performed excellent service to the industry and I know they are sincerely devoted to its welfare.

It seems apparent however, that in this instance the Editor allowed his zeal to transport him into the realm of fantasy. Incidentally, the statement that neither of the major Canadian airlines has bought Canadian equipment in the past decade does not happen to be true. Our own airline bought Canadian designed and built Otter aircraft in April 1955, for northern operations. Traditionally CPAL and its predecessor companies have always purchased Canadian aircraft and equipment whenever they were available and of a category to meet the requirement. Most of the Norseman and Fairchild aircraft in Canada were purchased and used by the companies which formed CPAL.

With the advent of the big multi-million dollar transports, however, the situation has changed radically and it seems obvious that the Canadian airlines could not support the design and production of Canadian airliners, even if they were to make such a commitment without regard for the economics involved.

The answer to this editorial, of course, lies in the question: What Canadian airliners have been offered to the airlines? In our own case, we did buy Canadair 4 airliners to pioneer our overseas routes, but when these became economically and competitively obsolete and when we placed orders for DC6B aircraft there was no competitive Canadian product on the market. Similarly in 1955 when we placed orders for our Britannia aircraft, there was no Canadian airliner available in this category. Obviously, of course, the same situation applied to TCA in the ordering of the big jets. Whether or not any of these aircraft for use by Canadian airlines could or should have been built in Canada was obviously a matter of air industry economics rather than airline policy.

I could recite an impressive list of Canadian products and services which we have purchased and are continuing to purchase but I do not wish to take up more of your time with the detailed discussion of this subject, and will only repeat my assurance that it is CPAL policy to buy Canadian whenever possible.

As we peer into the first decade of jet transportation, the prospect challenges the courage and confidence even of those with great faith in the air industry and its future.

Consider the fact that the world airlines have committed more than 6 billion dollars on a fleet of 1500 big jet and propeller turbine aircraft. Every one of these big jets has the capacity to carry more passengers back and forth across the Atlantic a year than the largest ocean liner. And it costs $3,000 a day to have one of these aircraft sitting idle on the ground. Somebody has to occupy all of these seat miles with revenue passengers if the airlines are to survive financially.

One cynical observer summed up the situation this way: "The airlines have bought jets they know nothing about, with money they don't have, to carry passengers they hope to get."

I disagree with these merchants of gloom. I am completely confident in the prospects of air transportation. One reason for my confidence derives from recognition of the technical capabilities of the aviation industry, which have been admirably demonstrated and which have been responsible for such miraculous progress since the war.

The most significant increases in air traffic have resulted from the introduction of lower fares and these have been made possible only by the technological advances which your aeronautical fraternity have made possible. The aeronautical engineers can take credit for the fast increases in the air transport market since the war.

My confidence in the future of this expanding industry is bolstered by a review of its recent history. International air transportation really was born — as a major industry — in 1945, when it rose spectacularly from the ashes of the last war. Some 9 million passengers flew on the newly created international routes in 1945. Such has been the astounding development of this industry that, compared with this original 9 million, more than 100 million passengers patronized the international airlines in 1960. And this is just a beginning!

Fostered by the incredible capacity of the big jets, and by the explosive growth of immigration, tourist and business transportation, the future of global air travel simply baffles the imagination.

Shifting of the world's population between continents has already become a very important traffic source in air transportation. This air movement will continue with rapid acceleration as the population

pressures increase and as the costs of air travel continue to improve in relation to surface transportation.

In Washington there is a population clock with a light that flashes every eleven seconds to mark the birth of another American. If there were such a world population clock, it would flash three times every second. Every year there are enough Chinese born to populate another Canada.

The present world population stands at about 2.6 billion people. It is an astonishing fact that of all the people who have ever lived on this earth, one-third are alive today! Such is the prospective population explosion that experts predict a world population of between 6 and 7 billion within 40 years.

To keep the Asian population stable, it would require the emigration of 25 million people a year. To give you some idea of such a flow of population, if these 25 million a year were flown to Canada in 100 passenger airliners, we would have to land from Asia one of these immigrant airliners every two minutes, or more than two per second night and day throughout the entire year!

This is not to suggest, of course, that there is any practical prospect of relieving the great population pressures of the world by mass aerial migration. It does suggest, however, that with the improving economies of vast areas, immigration, already a significant source of international air traffic, will assume an ever increasing importance in the economies of the airlines.

A more immediate and rewarding source of big jet air travel will result from the booming economies of North America, Western Europe, Japan, Mexico, Latin America and Australasia. Wherever you look in these countries you see the trend toward shorter working hours, longer paid vacations, generous pension plans – more money to spend and more time to spend it.

As one economist noted,

"The upward trend in standards of living in many countries is approaching the descending curve of airline fares, and when they meet there will be a tidal wave of tourism such as the world has never seen before."

Thus the jets with their speed, their intercontinental range and their economies will give a powerful impulse to the wave of international tourist travel.

Also related to these booming economies is the large and rapidly-mounting volume of international business travel. The traditional travelling salesman has become a world air traveller seeking customers and contracts many thousands of miles away, perhaps half way around the world from his home office but only hours away by jet.

Fortunately, Canadian businessmen are awakening to the fact that these once remote markets are now easily reached by air and offer rich rewards for energetic salesmanship.

The value of these international air routes in providing transport and communications for direct stimulation of Canada's foreign trade is obvious to any businessman. Do you realize, however, how important the Canadian air transportation industry is itself, as a producer and exporter of its product air service?

Canadian air services are shuttling passengers, most of them foreigners, between Hong Kong and Tokyo, Mexico City and Lima, Lima and Santiago, between Europe and Australia-New Zealand, and between Lisbon and Rome. This is in addition to the routes linking Canada's largest cities with countries on five continents.

This export of air transportation by CPAL has resulted in revenues exceeding 50 million dollars for Canada in the past ten years and this is just a fragment of what could have been earned if, like the Dutch for example, we had been sharp and enterprising enough to build up a truly great merchant marine of of the air. The Dutch airline, KLM, thriving almost entirely on its "Fifth Freedom" export-type traffic, earns about 150 million dollars a year for Holland. Developed on a corresponding scale, our own overseas air transport business could have been ten times as big as it was in the past decade. In other words, it could have been a source of some 500 million dollars in new revenue for Canada. When we consider that 40% of Canadian airlines' income is spent on wages this would have meant 200 million dollars in Canadian payrolls!

I am convinced that the big jets will create the new markets for domestic and international air transportation to occupy their vast new capacities. I consider as sound the forecast of Mr. Hees, formerly Minister of Transport, that in 15 to 20 years from now we will be carrying over 30 million domestic air passengers a year in Canada and over 10 million passengers to and from Canada every year on international flights. I do not doubt for a moment that the 100 million people a year who now fly the global air routes, will be 300 million or more within 10 years.

At this point I can imagine you saying to yourselves that all this sweeping optimism is fine but what does all this mean to the Canadian industry. This is a very good question. In fact, I believe it is one of the very important questions facing us today, because we have reached a point of decision. Either Canada must adopt an energetic, aggressive and enlightened air policy or our air industry will falter and fail in the race for competitive survival. We can become one of the great airfaring nations of the world, or we may be a third rate power in civil aviation. It all depends on the air policy our Government adopts and the energy with which the Canadian carriers exploit the opportunities unlocked by a new air policy.

I am firmly convinced that survival in the increasingly competitive arena of international air transportation will require the full energies of our Canadian air transport industry to be marshalled against foreign competition. The air policy must allow both carriers to exploit every opportunity to increase the Canadian share of international air traffic.

To achieve the maximum benefit for Canada, the air policy should allow the two prime carriers to operate under the same regulations without discrimination. They should compete where this is beneficial to the Canadian public, but should cooperate in other

instances, particularly in competition with foreign carriers. Traffic sharing arrangements should logically be made between the Canadian carriers first, and with foreign carriers only when such arrangements are clearly to the benefit of Canadian aviation.

In competing with non-Canadian carriers on routes between this country and foreign destinations, the Canadian carrier should have the natural advantage of access to the principal markets of this country. It is thus important that both Canadian international carriers be afforded ample opportunity to develop strong profitable domestic route patterns to tap the internal markets for overseas traffic and to provide a home market for export business as in other industries.

To be successful, Canadian air policy must allow the fullest freedoms of opportunity for Canadian carriers to compete, to find means of increasing the utilization of their aircraft fleets and to multiply the employment of Canadian personnel. Make no mistake, there are plenty of opportunities if we have the wits to discover and exploit them as our competitors are doing every day.

There is no lack of opportunity for a smart and enterprising businessman if free to exercise the privileges of competitive private enterprise.

Look at what the US operators are doing to us on the trans-border routes. They have us ham-strung, bound hand and foot to a bilateral agreement which is nothing more than a collection of tight little air monopolies shuttling back and forth across the border, forced to land and transfer the passengers on the other side of the line, regardless of their desires or destinations.

Can you imagine anything more ludicrous than the spectacle of two great free enterprise democracies conspiring to shackle and frustrate the natural flow of air traffic across their common border? Because of the selfish, short-sighted and restrictive policies imposed by the special interests which have influenced the bilateral agreement negotiations between our two countries, the passenger is either grounded at the border or in some cases has to fly trans-border on a foreign airline between major cities in Canada and the USA.

Much has been said about the famous "undefended border" extending nearly 4,000 miles from sea to sea, and recent reports indicate that 100 million travellers swarm across the invisible line between the USA and Canada every year. More and more of these travellers should be taking to the skies but the freedom and convenience of border crossing on the ground does not always extend into the air.

I am reminded of a ludicrous situation which existed for a short time in the early stages of the war, when US neutrality legislation decreed that military aircraft must not be flown across the border into Canada. However, there were men of vision in the USA who decided to take action. Aircraft were landed close to the border, a rope was flung across and the aircraft were towed into Canada and then flown onward to take part in the defence of democracy. We aren't hauling civil aircraft across the border but on some routes the restrictions are almost as inconvenient for the passenger and for the development of logical traffic patterns.

It is not right that passengers flying from Vancouver to San Francisco and Los Angeles should be forced to land at Seattle and change to another flight.

If we had a comparable situation in Europe there would be chaos, and air transportation would be reduced to a helpless shambles. It would mean, for example, that passengers from London to Paris would have to land at Calais and change to a French airline. I cannot think of any other border where such a ridiculous situation exists. Even Russia allows foreign airlines to penetrate to Moscow instead of forcing their passengers to land somewhere just inside the border and proceeding on a Russian airliner. Thus there are direct air services from London to Moscow, from Paris to Moscow, from Copenhagen to Moscow and from Amsterdam to Moscow.

It is an astounding fact that no Canadian or US carrier is allowed to operate directly between the largest city in Canada (Montreal) and the second largest city in the USA (Chicago), but a French airline is allowed to come in and carry Canadian passengers and US passengers directly between Montreal and Chicago!

This is also hard to believe but is a fact: There is no Canadian or American direct air service between two large Canadian and US cities on the West Coast – Vancouver and San Francisco. Yet an Australian airline is flying directly between Vancouver and San Francisco carrying Canadian and US passengers.

So here are we Canadians standing complacently on the side-line while the airline of a third country comes in and legally highgrades our own passenger traffic on these lucrative trans-border routes. We have no objection to competing with a foreign carrier for such business, in the same manner that we do in foreign countries where we enjoy Fifth Freedom rights. Between Canada and the USA, however, the foreign carriers can exercise their traffic rights while the airlines of the two countries concerned stand on the side-lines! I can think of no other place in the world where such a situation exists.

It is obvious that this situation cries out for an early solution.

The only logical approach is to decide which pairs of Canadian and American cities would justify direct air services if Canada and the USA were all one country. Would there be direct air service from Toronto to Los Angeles, yes; Toronto to San Francisco, yes; Vancouver to Los Angeles, yes; Vancouver to San Francisco, yes. It is obvious that the agreement between two countries should be worked out on this sensible basis. Then an airline designated by each country should have the opportunity of providing these logical services. These major routes could all be flown by carriers of both countries, and where traffic was not sufficient to support unlimited frequency there could be regulation of frequency such as prevails in certain other bilateral agreements throughout the world. The important thing is to pro-

vide the air services which the traveller has every right to expect.

As if this trans-border frustration were not galling enough to Canadians, we now witness the spectacle of the US carriers compounding the trans-border shuttles into long-range routes penetrating to the logical US destinations of Canadian travellers. For example, Eastern Airlines has Montreal-New York rights but is running big advertisements in Canadian papers featuring "Jets to Miami" direct from Montreal. Similarly Northeast Airlines, with traffic rights from Montreal supposedly to Boston, extends this to its Boston-Miami route to proclaim to Canadians "Fastest Jets to Miami". Northwest Airlines has a trans-border service Winnipeg to Fargo. By the same token this however becomes a route Winnipeg to Miami, Winnipeg to Chicago, and so on.

United Airlines supposedly has traffic rights Vancouver-Seattle, exchanged for TCA's local Vancouver-Victoria-Seattle run. But of course the US carriers have all to themselves the lucrative Canadian traffic from Vancouver to San Francisco and Los Angeles because they alone are allowed to carry it onward from Seattle.

Somehow we have allowed ourselves to be manoeuvred into the position of bumpkins at a county fair watching a slick but legitimate shell game. The money is going out of our pockets, the Canadian traffic is going to foreign competitors and we don't seem to be able to do anything about it.

The trouble is that we Canadians have not been aggressive enough in our negotiations with the USA. In the meantime other countries have risen up and have battled for their rights. When the US negotiators tried to block the Mexican airline at the border and to force them to dump their traffic at El Paso, they simply threatened to stop all American flights to Mexico. And so today, the Mexicans are flying right into Chicago, New York and Los Angeles from Mexico City.

Our US neighbours have opened their skies to many other countries but have slammed the door in the faces of their best customer and good neighbour, Canada. The French airline flies not only from Paris to Chicago and Los Angeles and from Mexico to New York but also operates between Montreal and Chicago. The Scandinavian airline is allowed to fly from Europe on its Polar route right into Los Angeles. The Australian airline not only flies directly between Vancouver and San Francisco but flies from San Francisco right across the United States to New York.

I believe there is reason for optimism as we survey the Canadian air industry's prospects. The policy developed by the Hon. George Hees as Minister of Transport to introduce airline competition gradually was a logical one.

The present Government is aware of the problems I have outlined. The Government is equally aware of the brilliant prospects for the air industry with the aid of an enlightened and enterprising air policy.

The aggressiveness of foreign carriers in draining off Canadian traffic to an alarming degree has drawn attention to the need for effective countermeasures. It seems probable that the authorities in Ottawa will apply to the air policy the same degree of energy and aggressiveness that has recently characterized the stimulation of our foreign trade.

I believe that the removal of the limitations which have hindered the full exercise of Canadian enterprise can assure a promising future for our air industry. I am confident that, competing on equal terms, our industry, including design and manufacture as well as transport, can match paces with any in the world. I can think of no more fitting description of this prospect that the famous RCAF motto, Per Ardua ad Astra — Through Adversity to the Stars.

CANADIAN AERONAUTICAL INSTITUTE

STABILITY OF LAMINAR FLOWS†

by Dr. D. W. Dunn*

National Aeronautical Establishment

INTRODUCTION

It has been recognized for nearly a century that in practical situations viscous shear flows do not remain laminar at high Reynolds numbers, even under apparently steady conditions, but become turbulent. Such flows include, for example, the fully-developed flow through channels and pipes, jets and the thin layers associated with a body in a stream to which significant viscous effects are restricted — the boundary layer and wake. The steady flows that are theoretically possible under steady conditions are replaced in practice by unsteady flows, in which the velocity components and the pressure fluctuate in a highly irregular manner, although they still possess steady mean values. For these turbulent fluid motions many of the quantities of direct physical interest, such as the surface shear stress of a boundary layer, have quite different values from the values for laminar motion.

Reynolds, who did experimental research on turbulent flow through pipes and channels as early as the latter part of the last century[1], was the first to recognize the fundamental significance in viscous flows of the non-dimensional parameter VL/ν, now known by his name, where V is a characteristic velocity of the flow, L a characteristic length, and ν the kinematic viscosity. Both Reynolds and Lord Rayleigh[2] formulated the hypothesis that the theoretically possible laminar flows become unstable at sufficiently large values of the Reynolds number, so that any small disturbances initially present (at the entrance to a pipe, for example) amplify in some manner and eventually lead to turbulent motion. Since such disturbances are in practice unavoidably present in the initial flow in pipes, channels and jets, and in the free stream flow of jets, wakes and boundary layers, the occurrence of turbulent motion at high Reynolds numbers is to be expected in most practical situations.

Since it has become apparent in recent years that the stability theory of laminar fluid motion may have more practical significance than previously thought, it seems worthwhile to bring some of the main results of the theory to the attention of a wider audience. Accordingly, the present article is intended as an introductory survey of the subject, with the main emphasis on topics of interest in engineering, particularly aeronautical, mechanical and civil engineering. For a detailed basic treatment, the reader is referred to the books by Lin[3] and Schlichting[4, 5]. Recent developments not touched on here are surveyed in several short articles by Lin[6, 7, 8], Schlichting[9], Serrin[10] and Görtler[11]. A comprehensive survey of the physical aspects of transition to turbulence and the experimental research up to the end of 1956 has been given recently by Dryden[12]. Shorter surveys emphasizing the physical and engineering aspects of stability and transition are those of Kuethe[13] and Morkovin[14]. Lin[3] includes a complete bibliography of the subject to the end of 1954, and the other authors give many of the important references from 1954 to the end of 1959. Although references to recent work are included in the present article, no attempt has been made to be complete in this respect because of limitations of space[a].

Although much of the theoretical research effort involves a great deal of complicated and often difficult mathematics, this aspect of the subject is avoided here as much as possible. However, in order to acquaint the reader with the nature of the theoretical approach and the standard terminology, the formulation of the mathematical problem is briefly outlined in a few simple cases.

OUTLINE OF THE EARLY HISTORY OF THE STABILITY THEORY

The concept of laminar flow instability has formed the basis of most research on the origin of turbulence. After the pioneering work of Reynolds and Rayleigh, many other research workers, including such well known theoretical physicists as Sommerfeld[15] and Heisenberg[16], were attracted to the problem of the stability of laminar motion. For several decades success was limited, but during the 1920's important contributions began to appear. In 1923, Taylor[17] developed a theory predicting instability at high Reynolds numbers for the viscous flow between rotating cylinders (Couette motion), and verified the theory experimentally. In 1924, Heisenberg[16] predicted the instability of the parallel flow in a channel (plane Poiseuille motion) at high Reynolds numbers, although he did not carry out detailed calculations. Prandtl and his co-workers at Göttingen did intensive research on the

†Paper extracted from the Quarterly Bulletin of the Division of Mechanical Engineering and the National Aeronautical Establishment, July to September, 1960 (DME/NAE 1960(3)).
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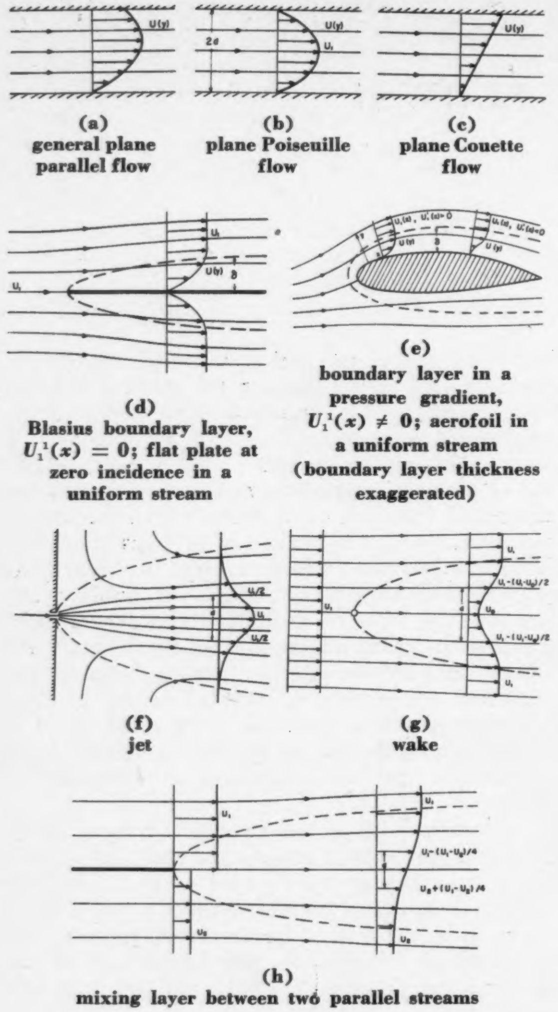
[a]It is interesting to note that Lin's book[3] includes 250 references to research on the stability theory and related subjects during the years 1875 to 1954, while a survey of the recent literature reveals over 300 such publications during the last 6 years.



(a) general plane parallel flow

(b) plane Poiseuille flow

(c) plane Couette flow

(d) Blasius boundary layer, $U_1^1(x) = 0$; flat plate at zero incidence in a uniform stream

(e) boundary layer in a pressure gradient, $U_1^1(x) \neq 0$; aerofoil in a uniform stream (boundary layer thickness exaggerated)

(f) jet

(g) wake

(h) mixing layer between two parallel streams

Figure 1
Examples of two-dimensional laminar flows

stability problem for the two-dimensional boundary layer over a flat plate at zero incidence in a uniform stream (Blasius flow). This led to Tollmien's successful theory of boundary layer stability in 1929[18], and later extensions of the theory by Schlichting[19, 20] and others in Germany during the period 1930 to 1945.

The main reason for the slow initial rate of progress in the stability theory of parallel flows (such as plane Poiseuille flow and, approximately, boundary layer flow) was the extraordinary mathematical difficulty of the theory. This also was the source of much controversy in the early results, and it was not until 1945 that the theoretical results on the stability of parallel flows were finally almost universally accepted. A comprehensive study of the theory was published at that time by Lin[21], who clarified the mathematical aspects and carried out calculations that verified in all essentials the earlier work of Heisenberg, Tollmien, and Schlichting. Also, information was released in 1945 on the experimental investigations of boundary layer stability completed during the war years by Dryden and co-workers Schubauer and Skramstad[22], who obtained experimental results in good agreement with the theoretical predictions. In contrast to the situation for parallel flows, the stability theory of curved flows did not present such difficult mathematical problems. Taylor's theoretical results[17] on the stability of the flow between rotating cylinders, although elaborate, were not controversial. His own experiments verified the theoretical results, and later theoretical investigations using different methods gave the same results.

Since 1945, many other research workers have continued, at a rapidly increasing rate, both experimental and theoretical research on the stability of fluid motion. Although many important results have been obtained, the subject is far from being closed, and still presents some of the most challenging problems in the whole field of fluid mechanics.

STABILITY OF PARALLEL FLOWS

In a viscous incompressible fluid, the most general type of two-dimensional parallel flow is one with a parabolic velocity profile. This is the flow between two parallel planes, one moving with respect to the other, with a pressure gradient in the flow direction (Figure 1a). The most important cases are *plane Poiseuille flow*, in which the two planes are fixed and there is a constant pressure gradient in the flow direction (Figure 1b), and *plane Couette flow*, in which one plane moves with respect to the other and the pressure is constant (Figure 1c). In addition, there are many nearly-parallel flows such as boundary layers (Figure 1) for which the stability theory is very similar.

The stability of a steady basic flow (u_0, v_0, w_0, p_0) is studied by considering it to be slightly disturbed, and then following the development of the disturbed flow with the time. Equations for the disturbances (u_1, v_1, w_1, p_1) in the velocity components and the pressure are obtained by subtracting the Navier-Stokes equations for the basic flow from the Navier-Stokes equations for the total flow (u, v, w, p), where $u = u_0 + u_1$, $\ldots$, $p = p_0 + p_1$[b]. The resulting disturbance equations are usually linearized by neglecting terms quadratic in the disturbances.

A complete solution of the initial value problem for the disturbances in the case of arbitrary initial conditions would in general be very complicated, and is usually avoided. A simpler and more informative approach is to consider particular solutions of such a form that a suitable superposition of them can satisfy arbitrary initial conditions. The stability characteristics for the general case then follow from those for the particular solutions. For example, in the case of a two-dimensional parallel flow with $u_0 = u_0(y)$ and $v_0 = w_0 = 0$, the partial differential equations for two-dimensional disturbances (u_1, v_1, p_1) have particular solutions of the form

[b]This is the conventional formulation. There are other ways of separating each of the total flow variables into two parts, although this is not always mentioned (e.g., see References 23, 24, p. 3, and 25).

$$u_1(x,y,t) = f(y)e^{i(\alpha x - \omega t)}, \quad (1)$$

plus similar expressions for v_1 and p_1 with amplitude functions $g(y)$ and $\pi(y)$ in place of $f(y)$. Such a solution corresponds to a plane wave progressing in the x-direction (Tollmien-Schlichting wave). The wave number α and frequency ω are taken to be real and complex, respectively, in the usual formulation, and $\omega = \omega_r + i\omega_i$. If $\omega_i > 0$ the basic flow is unstable, and if $\omega_i < 0$ it is stable. The restriction to two-dimensional disturbances is not serious, since Squire[28] showed that three-dimensional disturbances (u_1, v_1, w_1, p_1) of the form

$$u_1(x,y,z,t) = f(y)e^{i(\alpha x + \beta z - \omega t)} \quad (2)$$

are more stable. A solution of this type represents an oblique wave progressing in a direction at an angle arctan β/α to the x-direction. The three-dimensional problem is also very easily related to the two-dimensional one.

If expressions of the form (1) are substituted into the partial differential equations, ordinary differential equations in the disturbance amplitude functions $f(y)$, $g(y)$, $\pi(y)$ are obtained. From these, a single differential equation for $g(y)$ can be obtained by elimination;

$$(U - c)g'' - U''g = -\frac{i}{\alpha R}(g'''' - 2\alpha^2 g'' + \alpha^4 g) \quad (3)$$

where all quantities are non-dimensional. For example, the reference length and reference velocity for the non-dimensional quantities are the half-width d and maximum value of the velocity $u_0(y)$, in the case of plane Poiseuille flow. The wave speed c is defined by $c = \omega/\alpha$, R is the Reynolds number, and $U(y)$ is the basic-flow velocity profile. Eq. (3), which first appeared in the investigations of Orr[27] and Sommerfeld[15], is called the Orr-Sommerfeld equation. The boundary conditions are that $g(y) = g'(y) = 0$ at the boundaries of the flow (e.g., at $y = \pm 1$ in the case of plane Poiseuille flow).

The boundary conditions lead to an eigenvalue equation of the form

$$F(\alpha,R,c) = 0, \text{ or } c = G(\alpha,R) \quad (4)$$

Since $c = c_r + ic_i$ is a complex constant, Eq. (4) is equivalent to two real equations

$$c_r = G_r(\alpha,R) \text{ and } c_i = G_i(\alpha,R) \quad (5)$$

The exact functional forms of the functions F and G depend on the particular problem considered. Putting $c_i = 0$ gives the curve of neutral stability in the (R, α) plane.

In all stability investigations, the formulation of the mathematical problem is very much like that just described. The basic flow, although it can in general be three-dimensional, is restricted to being two-dimensional ($w_0 = 0$) or axisymmetric, or subject to some other symmetry condition, in most existing work, and will be so considered here unless otherwise specified. The disturbance flow may be either two- or three-dimensional. The disturbance quantities (u_1, v_1, w_1, p_1) corresponding to Eq. (2) are complex, but may be added to their complex conjugates to give real quantities representing a physical disturbance. Although such a simple oscillatory disturbance plays the dominant role in all existing theoretical work, it should be emphasized that the linear theory allows much more complicated, non-periodic disturbances because of the possibility of superposition of particular solutions.

The actual calculations of the eigenvalues in the case of a particular velocity profile $U(y)$ are very elaborate, and are usually based on asymptotic approximations to four linearly-independent solutions of Eq. (3) for large values of αR (since instability is expected only for large Reynolds numbers in most cases). Finding suitable asymptotic solutions is in general a very difficult mathematical problem, and was the source of the difficulties and controversies in the early investigations of the stability of parallel flows.

A detailed discussion of the mathematical aspects of the theory will not be given here, but two points are worth noting. First, the asymptotic solutions are very sensitive to $U''(y)$, so that the stability characteristics are very sensitive to variations in the velocity profile. Secondly, the vicinity of the critical point where $U(y) = c$ plays an important role in the mathematical theory of the asymptotic solutions. Physically, this is reflected in the fact that for neutral disturbances, no matter how large the Reynolds number, viscous effects are always important in a thin *critical layer* in the fluid where the local fluid velocity is equal to the wave speed[c]. For damped disturbances, moreover, this viscous layer about the critical point may have a thickness comparable to the width of the basic laminar flow. Lin's book[3] discusses these points in great detail.

Although viscosity evidently plays an important and complex role in the stability problem, a few useful results can be obtained by neglecting it, that is by omitting the terms on the right hand side of Eq. (3). Most of the early investigations were based on this inviscid approximation. Also, since any parallel flow is possible in an inviscid fluid, the velocity profile $U(y)$ is not restricted to the parabolic form described previously but can be completely general[d]. Rayleigh[2] obtained several results for the inviscid case, the most interesting one being that a necessary condition for the existence of unstable disturbances is the presence of an inflection point in the velocity profile. Tollmien[25] later proved that this is a sufficient as well as necessary condition for instability.

The inflection point criterion of the inviscid theory cannot be completely extended to the viscous problem. It is true that the presence of an inflection point in a velocity profile is destabilizing, but the absence of one does not necessarily imply complete stability. If it did, plane Poiseuille flow would be completely stable, since $U''(y)$ is always negative. Even though this agrees with the intuitive notion that viscosity has a damping influence on disturbances, de-

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[c]For neutral disturbances, the theory shows that $0 < c < 1$ when $0 < U(y) < 1$, so that such a layer always exists.

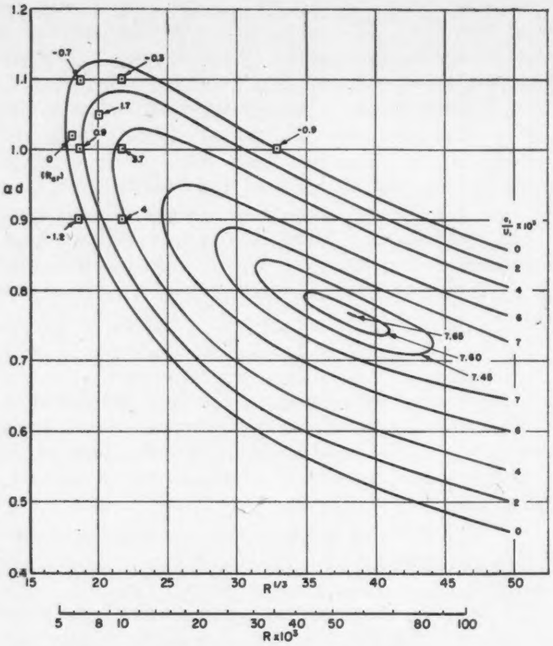
[d]Thus the results are useful in the inviscid limit for nearly-parallel flows, such as boundary layers.

tailed calculations based on the full Orr-Sommerfeld equation show that instability does occur. Thus, viscosity actually causes the instability of plane Poiseuille flow.

The first consideration of the complete Orr-Sommerfeld equation was the investigation of plane Couette flow by Sommerfeld[15], who found no evidence of instability at any Reynolds number. This conclusion was verified in all later investigations, the latest being that of Wasow[29]. Since turbulent plane Couette flow occurs experimentally, several suggestions for the occurrence of instability and transition have been made. A possibility suggested by Schlichting[30] is that transition results as a consequence of instability of the unsteady flow preceding the steady flow in practice. Another possibility is that instability is a result of non-linear effects neglected in the theory.

Heisenberg[16] predicted that plane Poiseuille flow is unstable at sufficiently high Reynolds numbers, but did not carry out detailed calculations. Later investigators questioned his methods and conclusions, and claimed to demonstrate complete stability at all Reynolds numbers. However, Lin[21] clarified the mathematical basis of the theory, and by using the procedure outlined by Heisenberg completed detailed calculations of the neutral curve, which verified Heisenberg's prediction. Shen[31] later repeated these calculations, and also computed curves of constant amplification c_i. The results are shown in Figure 2. To each pair of values (α, R) on any of these curves there is a corresponding value of c_r in the range $0 \leq c_r < 1$. The region enclosed by the curve of neutral stability ($c_i = 0$) corresponds to unstable disturbances, while the outside region corresponds to stable ones. Both the upper and lower branches of the neutral curve approach the R axis ($\alpha = 0$) when R becomes infinite, in conformity with Rayleigh's conclusion that no unstable disturbances can exist in the inviscid limit when the velocity profile has no inflection point.

In addition to the plane parallel flows considered so far, there are also corresponding axisymmetric parallel flows, such as the *Poiseuille flow* through a long circular tube. The stability investigation of these flows is similar to that already described, and is usually limited to disturbances that are rotationally symmetric (i.e., independent of the angle about the axis). All investigations of Poiseuille flow that have been carried out so far show no evidence of instability at any Reynolds number. The latest theoretical investigations are those of Sexl and Spielberg[32], Corcos and Sellars[33], and Spielberg and Timan[34]. In the latter investigation, some aspects of the problem in the case of general three-dimensional disturbances are considered, and it is shown that there is no simple analogue of Squire's theorem relating these disturbances to the simpler rotationally symmetric disturbances usually considered. The transition to turbulence observed experimentally might be due to instability of the entrance flow, as suggested by a theory of Tatsumi[35], three-dimensional effects, or possibly to non-linear effects, as suggested by the recent experimental results of Leite[36].



POINTS COMPUTED BY THOMAS, REF. 105. NUMBERS ARE VALUES OF $C_i/U_1 \times 10^3$

Figure 2

Stability characteristics of plane Poiseuille flow; $R = U_1 d/\nu$, U_1 = maximum velocity, $2d$ = channel width (after Shen[31] and Thomas[105])

In addition to the conventional formulation of the theory, with α and β real and ω complex in Eq. (2), there is a modified formulation with β and ω real and α complex. This modified formulation, which does not seem to be well known, would appear to have more physical significance in the case of parallel laminar flows*. For such flows, instability and transition are recognized in practice by the growth of disturbances with the distance x in the flow direction. The disturbance amplitude remains constant at any fixed location in the flow. Thus, the question of interest concerns spatial stability rather than temporal stability, as in the conventional approach. Brief references to the modified formulation have been made by Battin and Lin[37], Corcos and Sellars[33] and Reshotko[38]. Since the case of neutral stability is obviously the same in either approach, the quantities α_i and ω_i are related under certain conditions when each is small. Battin[39] has derived the relations and discussed their significance. A fair approximation in some cases is $\omega_i = -c_r \alpha_i$, but such a simple relation is not valid in general. No curves of constant α_i have been computed as yet, to the author's knowledge, but the procedure would seem to be straightforward. Such results would correspond directly to the amplification factor measured in most experiments.

*The conventional approach might be the most appropriate in the case of flows with closed streamlines, as discussed in the next section.

STABILITY OF FLOWS WITH CURVED STREAMLINES

The simplest type of fluid motion with curved streamlines is the flow between concentric rotating cylinders (Couette flow). This flow is given by an exact solution of the Navier-Stokes equations for a viscous incompressible fluid expressed in cylindrical co-ordinates (r, ϕ, z), where z is the distance along the axis of rotation, r the distance from the axis, and ϕ the angle about the axis. The velocity components in the directions of increasing (r, ϕ, z) are here taken to be (v, u, w).

If R_1 and R_2 are the radii of the inner and outer cylinders and Ω_1 and Ω_2 their angular velocities, the solution for Couette motion is given by

$$U = r\left(A + \frac{B}{r^2}\right), \quad V = W = 0 \qquad (6)$$

where all quantities are non-dimensional. The parameters A and B are simple functions of R_2/R_1 and Ω_2/Ω_1 whose explicit forms depend on the particular reference length and reference velocity that have been selected.

In the same manner as described previously, the stability problem is defined and linearized equations for small disturbances (u_1, v_1, w_1, p_1) are derived. Since the basic-flow velocity components are of the form $u_0 = U(r)$, $v_0 = w_0 = 0$, the disturbance partial differential equations allow particular solutions for three-dimensional disturbances of the form

$$u_1(r,\phi,z,t) = f(r)\exp\left[i(n\phi + \beta z - \omega t)\right], \qquad (7)$$

plus similar expressions for v_1, w_1, and p_1, with amplitude functions $g(r)$, $h(r)$, and $\pi(r)$ replacing $f(r)$. The quantities n and β are taken to be real, and n must in fact be an integer, since the fluid motion must obviously be periodic in ϕ. The quantity ω may be complex in general. It does not appear that a modified formulation, as described in the last section, is possible in this problem. In addition to periodicity in ϕ, the boundedness of the disturbances for large values of z seems to be physically necessary. Thus, n and β must both be real. In all existing work n is taken to be zero, that is, only the case of rotational symmetry is considered. Physically, elementary disturbances of the form (7) with $n = 0$ represent ring vortices.

The non-dimensional equations for the disturbance amplitude functions can be reduced to:

$$(L - \beta^2 - \sigma R)f = R\frac{1}{r}\frac{d}{dr}(rU)g \qquad (8)$$

$$(L - \beta^2 - \sigma R)(L - \beta^2)g = 2\beta^2 R\frac{U}{r}f \qquad (9)$$

where $\sigma = -i\omega$, R is the Reynolds number, and L is the operator

$$L = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} = \frac{d}{dr}\left(\frac{d}{dr} + \frac{1}{r}\right) \qquad (10)$$

Eqs. (8) and (9) are to be solved subject to the boundary conditions

$$f = g = \frac{dg}{dr} = 0 \text{ for } r = r_1 \text{ and } r = r_2, \qquad (11)$$

where r_1 and r_2 are the non-dimensional radii of the inner and outer cylinders[f].

The resulting eigenvalue problem leads to an eigenvalue equation of the form

$$F(\sigma,\beta,R_2/R_1,\Omega_2/\Omega_1,R) = 0 \qquad (12)$$

For neutral stability $\sigma = 0$ (i.e., constant amplitude oscillations with σ purely imaginary are impossible), and for a given basic flow R_2/R_1 and Ω_2/Ω_1 are given, so that the neutral curve in the (R, β) plane is given by a relation of the form

$$R = R(\beta,R_2/R_1,\Omega_2/\Omega_1) \qquad (13)$$

The minimum critical Reynolds number,

$$R_c = R_c(R_2/R_1,\Omega_2/\Omega_1) \qquad (14)$$

is then obtained for a definite wave-number

$$\beta_c = \beta_c(R_2/R_1,\Omega_2/\Omega_1) \qquad (15)$$

Analytical simplifications result from approximations that are possible when R_1 and R_2 are nearly equal. In this case, $R_2 - R_1$ and $\Omega_1 R_1$ are suitable as a reference length and reference velocity respectively, and instead of r the variable y is introduced, where $r = y + R_1/(R_2 - R_1)$ and $0 \leq y \leq 1$.

With the neglect of terms $0(R_2 - R_1)/R_1$, Eqs. (8), (9) and (11) become

$$(D^2 - \beta^2 - \sigma R)f = R\frac{dU}{dy}g \qquad (16)$$

$$(D^2 - \beta^2 - \sigma R)(D^2 - \beta^2)g = 2\beta^2 R\frac{R_2 - R_1}{R_1}Uf \qquad (17)$$

with boundary conditions

$$f = g = Dg = 0 \text{ at } y = 0 \text{ and } y = 1, \qquad (18)$$

where $R = \Omega_1 R_1(R_2 - R_1)/\nu$ and $D = d/dy$.

The velocity profile of Eq. (6) reduces to

$$U(y) = 1 + \left(\frac{\Omega_2}{\Omega_1} - 1\right)y + 0\left(\frac{R_2 - R_1}{R_1}\right) \qquad (19)$$

in this case, so that Eqs. (16), (17) and (18) are, to the same order of approximation, equivalent to the single equation

$$(D^2 - \beta^2 - \sigma R)^2(D^2 - \beta^2)f = -\beta^2 T(1 + ky)f, \qquad (20)$$

with boundary conditions

$$f = (D^2 - \beta^2 - \sigma R)f = D(D^2 - \beta^2 - \sigma R)f = 0$$
$$\text{at } y = 0 \text{ and } y = 1, \qquad (21)$$

where $k = (\Omega_2/\Omega_1) - 1$, and $T = -2R^2k(R_2 - R_1)/R_1$. Most theoretical investigations have been restricted to this approximate formulation.

The approximate eigenvalue problem involves fewer parameters than the exact one. In fact, when the cylinders rotate in the same direction, approximating $(1 + ky)$ by an average $\frac{1}{2}(1 + \Omega_2/\Omega_1)$ is often accurate enough, so that when the additional factor is

[f]The explicit definitions of the parameters R, r_1 and r_2 depend on the reference length and reference velocity selected for the non-dimensional variables.

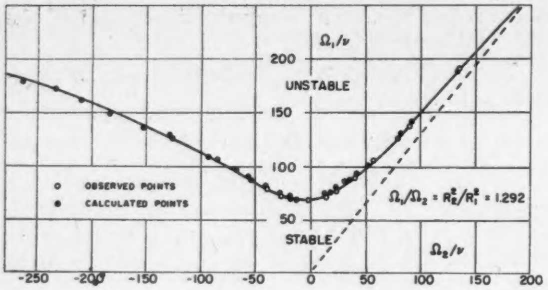


Figure 3

Stability characteristics of Couette motion; R_1 and R_2 are, respectively, the radii of the inner and outer cylinders, Ω_1 and Ω_2 are their angular speeds (after Taylor[17])

absorbed in the definition of T the eigenvalue equation has the simple form

$$T = T(\beta,\sigma) \tag{22}$$

Thus, for neutral stability both β and T have constant numerical values, which are given by detailed calculations as

$$T = 1709 \text{ and } \beta = 3.12 \tag{23}$$

Taylor[17] carried out calculations for cylinders rotating in both the same and opposite directions, and the theoretical results are shown in Figure 3 along with the experimental results.

In contrast to the situation for parallel flows, the mathematical difficulties of the stability theory of curved flows are not as great. Asymptotic methods involving intricate and controversial mathematical questions can be avoided, and satisfactory results obtained by relatively straightforward approaches. Taylor's calculations, although elaborate, were not controversial, and agreed very well with his own experimental results and with the calculations of later investigators (e.g., see Reference 3, pages 22 to 26). Chandrasekhar and Reid[40, 41] have recently developed more refined procedures which give accurate results with less computational labour than the earlier methods. Chandrasekhar has also considered the exact problem when the difference of the two radii is not small[42]. Di Prima[43] has generalized the problem to include a pressure gradient acting around the cylinders and along the streamlines.

The role of viscosity in the present problem is also simpler than in the stability theory of parallel flows. As one would expect physically, viscosity always has a stabilizing effect, so that the modifications required for any results from inviscid theory are obvious. Rayleigh[8] and later investigators developed several useful results for the stability of inviscid flows with curved streamlines (see Reference 3, pages 49 to 52). Very simple considerations neglecting viscosity show, in fact, that the centrifugal force is the dominating factor, and that a necessary and sufficient condition for stability is $d/dr\,[(\omega r^2)^2] > 0$, where r is the distance from the centre of curvature and ω the angular velocity of a fluid particle. In the case of Couette flow, this is equivalent to the condition $\Omega_2 R_2^2 > \Omega_1 R_1^2$. The inviscid limit of stability $\Omega_1/\Omega_2 = R_2^2/R_1^2$ is shown in Figure 3, along with the viscous limit computed by Taylor.

Another flow with curved streamlines having similar three-dimensional stability characteristics is the flow in a curved channel under the action of a pressure gradient in the flow direction. If the length of the channel and radius of curvature of its walls are both much greater than its diameter, the flow is represented by another exact solution of the Navier-Stokes equations, except near the ends. The streamlines are parallel to the curved walls, and to a close approximation the pressure gradient is constant and directed along the streamlines, while the velocity profile is parabolic, as in plane Poiseuille flow.

Consideration of three-dimensional elementary disturbances of the form (7) with $n = 0$ (representing vortices with their axes in the flow direction) again leads to disturbance equations similar to (16) and (17), where $R_2 - R_1$ is replaced by the channel diameter d, R_1 is the radius of curvature of the channel, R is the Reynolds number $u_m d/\nu$, and $U(y)$ is the parabolic velocity distribution referred to the maximum velocity u_m. The eigenvalue problem leads to an eigenvalue equation of the form

$$F(\sigma,\beta,R\sqrt{d/R_1}) = 0 \tag{24}$$

Dean[44] was the first to investigate this problem, and calculated a curve of neutral stability ($\sigma = 0$) with the minimum critical value $R\sqrt{d/R_1} = 54.0$ at $\beta = 3.95$. Yih and Sangster[45] recently considered the same problem, obtaining different numerical results, apparently erroneously according to Hämmerlin[46] and Reid[47]. Both Hämmerlin and Reid obtain very nearly identical results to Dean's by the use of different methods of solution.

It is interesting to note that even for d/R_1 as small as 10^{-4}, which obviously corresponds to a very small curvature, the minimum critical Reynolds number is $R = 5400$, which is less than the minimum critical Reynolds number $R = 10600$ for the flow in a straight channel in the case of two-dimensional disturbances (Figure 2)[e]. Thus, in practice, a channel would have to be very straight indeed before the three-dimensional instability considered here is dominated by the two-dimensional instability described previously.

An alternative formulation similar to that discussed previously, in which spatial stability is involved rather than temporal stability, is mentioned in Dean's paper[44], although no calculations beyond the case of neutral stability, which is obviously the same in either formulation, are actually carried out. In this formulation, ω in Eq. (7) is real and $n\phi$ is replaced by αx, where α is complex and $x = R_1\phi$ is the distance in the flow direction. Also, it appears that α_r should be zero for the class of elementary disturbances of interest here, which correspond to vortices with their axes in the flow direction. Since such disturbances are evidently a special class of the more general three-dimensional disturbances defined by Eq. (2), it is clear that Squire's

[e]Note that the Reynolds number in Figure 2 is based on half the channel diameter.

theorem requires reconsideration in the case of laminar flows over curved boundaries.

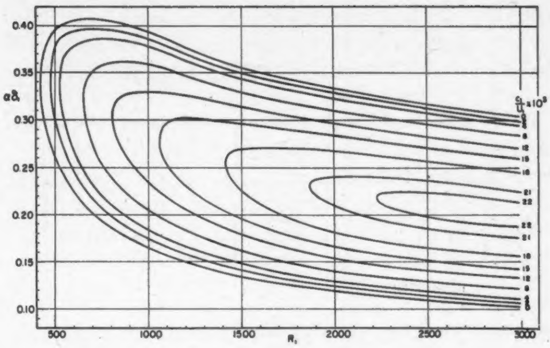
It is to be noted that eigenvalue problems mathematically similar to the ones described in this section occur in many other physical problems in the stability of fluid motion. Examples include the thermal stability of a horizontal layer of fluid with a vertical temperature gradient in a gravitational field, and various stability problems in magneto fluid dynamics (see Reference 3, pages 106 to 114, and Reference 40). A forthcoming book by Chandrasekhar[48] contains an exhaustive treatment of such problems and methods of solution.

STABILITY OF BOUNDARY LAYERS AND OTHER NEARLY-PARALLEL FLOWS

In addition to the exactly parallel flows considered previously, there are several flows that are nearly parallel from the point of view of the stability theory; namely, the boundary layer along a solid wall, and free boundary layers such as the jet, the wake and the mixing layer between parallel streams (Figure 1). The stability investigation of such flows can be carried out by relatively simple extensions of the procedures already described, if certain approximations are accepted.

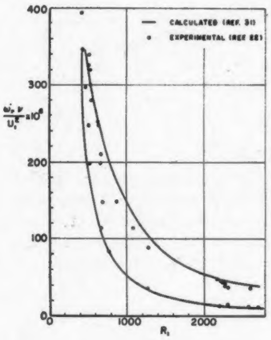
The stability of the boundary layer along a solid wall has been extensively studied, both theoretically and experimentally. The experimental investigations are easier for boundary layer flows, in fact, than for the theoretically simpler flow in a channel. In the theoretical study of two-dimensional stability, one proceeds by considering the local stability of the flow at a definite distance x from the leading edge with respect to disturbances that are essentially as described by Eq. (1). The analysis again leads to an eigenvalue problem for the Orr-Sommerfeld Eq. (3), where $U(y)$ is now the velocity profile of the boundary layer at the location x. The reference quantities are now the local free-stream velocity $U_1(x)$ at the edge of the boundary layer and the boundary layer thickness $\delta(x)$. The approximations involve the neglect of the component $V(y)$ of the basic-flow velocity normal to the wall and the neglect of the x-dependence of the basic-flow quantities. Pretsch[49] made an elaborate analysis of the errors involved in these approximations and showed that they are no larger than the errors present in the usual asymptotic methods of solution of such problems. Since instability normally occurs at large Reynolds numbers, just as for plane Poiseuille flow, the asymptotic methods developed for the latter problem are also appropriate here.

Tollmien[18] was the first to calculate the complete curve of neutral stability for the Blasius boundary layer on a flat plate, and Schlichting[19, 20] extended the calculations to include amplitude distributions and curves of constant amplification. Lin[21] repeated the calculations by a different method, the one suggested by Heisenberg[16], and verified the earlier results in all essential respects. More accurate calculations were carried out by Shen[31], and the results are shown in Figure 4 together with some of the experimental results of Schubauer and Skramstad[22]. It is to be noted



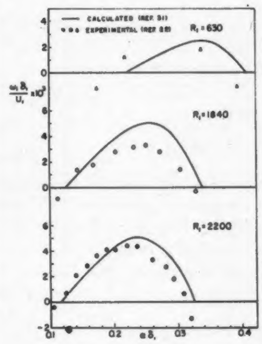
(a)

Curve of neutral stability ($c_i = 0$) and amplification curves ($c_i \neq 0$) for the Blasius boundary layer; $R_1 = U_1\,\delta_1/\nu$, U_1 = free-stream velocity, δ_1 = displacement thicknesss (after Shen[31])



(b)

Curve of neutral stability for the Blasius boundary layer in terms of frequency (after Shen[31], and Schubauer and Skramstad[22])



(c)

Amplification rates for the Blasius boundary layer (after Shen[31], and Schubauer and Skramstad[22])

Figure 4

that the disturbances that first become unstable (at $R_1 \approx 420$ and $\alpha\delta_1 \approx 0.33$) have a wave length rather large compared with the boundary layer thickness, namely $2\pi/\alpha \approx 19\delta_1$, where δ_1 is the displacement thickness. Both the upper and lower branches of the neutral curve approach the R_1 axis as R_1 becomes infinite, since the velocity profile has no inflection point. The discrepancies between the theoretical and experimental amplification curves are probably due to the inaccuracy of Shen's calculations plus the use of the approximate relation $\omega_i = -c_r\alpha_i$ by Schubauer and Skramstad in reducing their data (see last paragraph under the heading "Stability of Parallel Flows").

The two-dimensional stability characteristics of any boundary layer follow from the Orr-Sommerfeld equation to the same order of approximation as those for the Blasius boundary layer. The effects of such external parameters as the free-stream pressure gradient and the rate of fluid injection or suction at the wall enter only indirectly, through $U(y)$. Because of the very strong dependence of the eigenvalues on

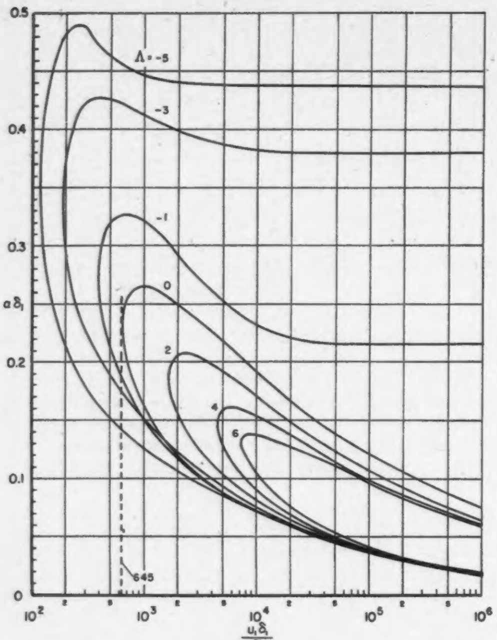


Figure 5

Curves of neutral stability for boundary layers with pressure decrease ($\Lambda > 0$) and pressure increase ($\Lambda < 0$); $\Lambda = \delta^2 U_1'(x)/\nu$, $U_1(x)$ = local free-stream velocity at edge of boundary layer, δ_1 = displacement thickness = $\delta(3/10 - \Lambda/120)$ (after Schlichting and Ulrich[50])

the second derivative $U''(y)$, small variations in the external parameters often have a large influence on boundary layer stability. Also, Rayleigh's criterion indicates that any factor causing an inflection point in the velocity profile leads to instability at infinite Reynolds number. Detailed calculations show that even at finite Reynolds numbers the presence of an inflection point is destabilizing.

Calculations of the effect of a pressure gradient on the stability characteristics of a boundary layer were made by Schlichting and Ulrich[50], with the results shown in Figure 5. Later results based on somewhat different methods are not significantly different. The latest are those of Zaat[51]. The effect of suction at the wall is shown in Figure 6 from the calculations of Ulrich[52]. The highly destabilizing effect of an adverse pressure gradient ($\delta^2 U_1'(x)/\nu < 0$) is to be noted in Figure 5 and, in particular, the finite range of unstable wave numbers in the limit of infinite Reynolds number corresponding to the presence of an inflection point in the velocity profile. Fluid injection has a similar destabilizing influence. On the other hand, the strong stabilizing effect of a relatively small favourable pressure gradient or positive suction velocity is clear from Figures 5 and 6. Other applications of the stability theory are described in Schlichting's book[4], which contains a comprehensive summary of the early German work.

The so-called free boundary layers, for which solid boundaries are absent, are another class of nearly-parallel flows whose stability characteristics can be determined approximately from the Orr-Sommerfeld equation. The simplest examples are the two-dimensional and axisymmetric jet, wake and mixing layer between two parallel streams of different velocities (Figure 1). These flows have velocity distributions extending to infinity in any direction normal to the main flow direction, with at least one inflection point in the profile. Rayleigh's criterion suggests at once that such flows are highly unstable, and this is in fact verified by detailed calculations. In every case there is a finite range of unstable wave numbers for infinite Reynolds number and a very small minimum critical Reynolds number.

The stability of free boundary layers in the inviscid limit was first studied by Hollingdale[53], Savic[54], and Lessen[55], in the case of the two-dimensional wake, jet and mixing layer, respectively. Lessen also obtained some results for large but finite Reynolds numbers on the upper branch of the neutral curve, using an asymptotic method similar to one used previously in the case of flows with solid boundaries. However, such asymptotic methods are appropriate only at large Reynolds numbers, and fail to give useful results at low Reynolds numbers and on the lower branch of the neutral curve.

Satisfactory results for low Reynolds numbers require different analytical approaches and have only been obtained within the last few years. Tatsumi and Kakutani[56] have used solutions of the Orr-Sommerfeld equation in the form of series in ascending powers of αR to calculate the lower branch of the neutral curve and a minimum critical Reynolds number for the two-dimensional jet. The upper branch at large Reynolds numbers follows from asymptotic expansions in inverse powers of αR, of the type used in earlier investigations. Although the calculations have not been carried far enough to give the entire neutral curve in the (α, R) plane, they indicate the general shape shown in Figure 7. In marked contrast to the situation for fluid motion near solid boundaries, the minimum critical Reynolds number is very small, and the range of unstable wave numbers decreases with decreasing Reynolds number, so that viscosity always has a stabilizing effect. Howard[57] independently obtained similar results for the jet.

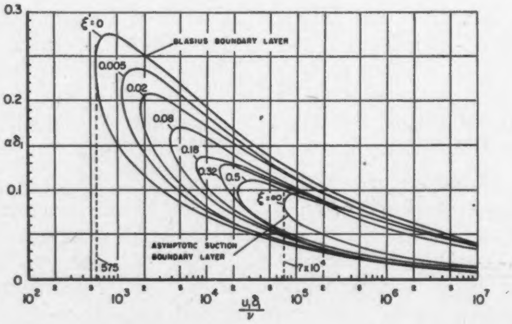


Figure 6

Curves of neutral stability for the boundary layer on a flat plate with uniform suction; $\xi = (-v_0/U_1)^2\, U_1 x/\nu$, U_1 = free-stream velocity, v_0 = suction velocity δ_1 = diplacement thickness (after Ulrich[52])

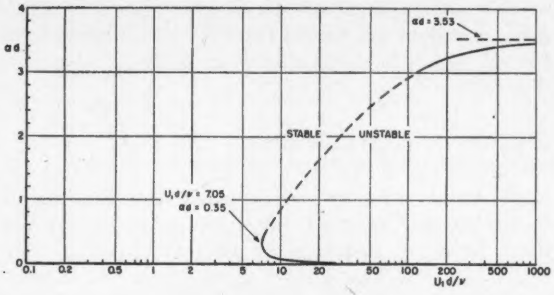


Figure 7

Curve of neutral stability for the two-dimensional laminar jet; U_1 = maximum velocity, d = velocity half-width as in Figure 1 (after Tatumi and Kakutani[50])

In the case of the wake, such complete results have not yet been obtained. Curle's calculations[58] may not be reliable, because of the objections to the method pointed out in References 56 and 57 and in Lin's review[59], in regard to Curle's investigation of the jet[60]. Arkhipov[61] has investigated the stability of the two-dimensional wake by means of the Galerkin method. However, since the calculations include no convergence studies and use only a two-term approximation for the eigenfunction, a check on the accuracy by other methods would be useful.

Tatsumi and Gotoh[62] have recently applied the method used previously for the jet to the stability problem for the two-dimensional mixing layer between two parallel streams, with the results shown in Figure 8. Again, the entire neutral curve is not obtained, but it is clear that the minimum critical Reynolds number is zero and that the lower branch of the neutral curve probably coincides with the R axis. Although these results are no doubt a logical consequence of the assumptions made, they contradict the intuitive notion that viscosity is always stabilizing at sufficiently small but finite Reynolds numbers. However, Tatsumi and Gotoh, as well as previous investigators, have pointed out that in the stability theory of free boundary layers there is a serious objection to the assumption that the flow is nearly parallel. Although the assumption is certainly valid at large Reynolds numbers, it is not in general valid at the small Reynolds numbers for which instability first occurs in such flows. For this reason, results obtained by assuming that the flow is essentially parallel may be physically unrealistic at small Reynolds numbers.

Very little has been done on the stability problem for axisymmetric free boundary layers, to the author's knowledge, except for some limited results on the axisymmetric jet by Lew[63], and calculations of the critical Reynolds number for the axisymmetric wake behind a sphere by Kawaguti[64, 65]. The latter investigation is another interesting example of the application of the Galerkin method.

In the case of two-dimensional flows with curved streamlines, the previous discussion of some simple flows of this type indicated that they could be unstable with respect to a class of three-dimensional disturbances representing vortices with their axes along the main-flow streamlines. The more complicated flows considered in this section also have streamlines with pronounced curvature in some cases, an important example being the boundary layer over a curved wall. The effect of this curvature on the stability characteristics is often of importance.

In the case of two-dimensional disturbances of the type described by Eq. (1), Görtler[66] showed that curvature of the wall has only a negligible direct influence on the boundary layer stability characteristics, so that the Orr-Sommerfeld equation is still an adequate basis for calculations if $U(y)$ is taken to be the local distribution of velocity parallel to the surface. The curvature may have an indirect effect through $U(y)$, of course, if the variation of the local free-stream velocity $U_1(x)$ at the edge of the boundary layer is due to the shape of the body, as in the case of an aerofoil in a uniform stream, for example[h].

When three-dimensional disturbances similar to those defined by Eq. (2) are considered, a detailed study of the problem shows that in the limiting case $\alpha_r = 0$ (when the disturbances correspond to vortices in line with the main flow) curvature terms must be explicitly considered in the disturbance equations. The problem turns out to be very much like the stability problem for the curved channel discussed previously, and the basic disturbance equations are exactly the same after certain approximations are made, if $U(y)$ is taken to be the local boundary layer velocity distribution, $\delta(x)$ the reference length, and $U_1(x)$ the reference velocity. The approximations are the usual ones employed in boundary layer stability investigations, which neglect the velocity component $V(y)$ of the basic flow and the x-dependence of the basic flow quantities.

Görtler[67] was the first to study this vortex instability of boundary layers, and obtained the main results. In the case of a wall that is concave outwards, the boundary layer is unstable when the parameter $R_\theta\sqrt{\theta/r}$ is large enough, where r, θ, and R_θ are the local radius of curvature, momentum thickness and Reynolds number based on momentum thickness, respectively. Also, the numerical results are relatively insensitive to the shape of the velocity distribution

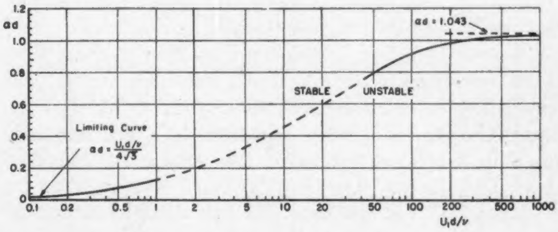


Figure 8

Curve of neutral stability for the two-dimensional laminar mixing layer between two parallel streams with velocities $U_1 \neq 0$ and $U_2 = 0$; d = velocity half-width as in Figure 1 (after Tatsumi and Gotoh[62], and Lessen[55])

[h] $U_1'(x) \neq 0$ in some cases when the surface is flat, e.g., the entrance region of convergent and divergent channels, and a flat plate at incidence or a wedge in a uniform stream.

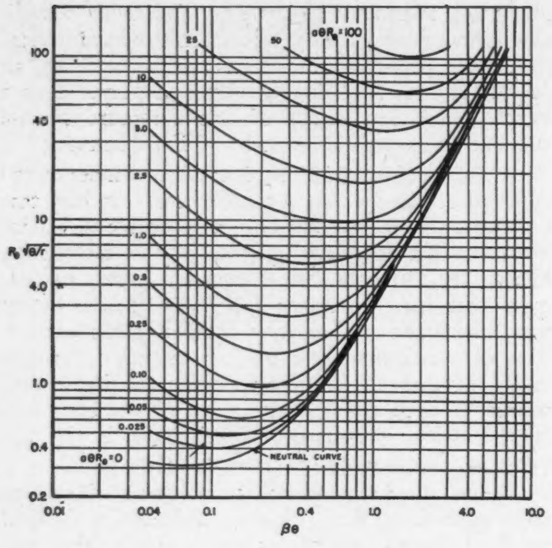


Figure 9

Stability characteristics of Taylor-Görtler vortices in the Blasius boundary layer over a concave surface; $R_\theta = U_1\theta/\nu$, U_1 = free-stream velocity, θ = momentum thickness, δ_1 = displacement thickness = 2.59θ, r = surface radius of curvature, a = amplification parameter (after Smith[68])

and, therefore, to any parameter that affects this distribution, such as the free-stream pressure gradient. The last conclusion is in marked contrast to the situation for the two-dimensional instability considered previously. Smith[68] later carried out more extensive calculations, using a formulation involving disturbance amplification in x rather than t, similar to the one suggested earlier by Dean[64]. The resulting curves of constant amplification in the case of the Blasius velocity distribution[1] are shown in Figure 9, where $a = -\alpha_i$ is the amplification parameter corresponding to a factor $\exp\left(-\int_0^x \alpha_i dx\right)$ in the definition of the disturbance[2]. Since $\beta\theta$ at the minimum value of $R_\theta\sqrt{\theta/r}$ is 0.1, the disturbances that first become unstable have a wave length $2\pi/\beta = 63\theta \approx 24\delta_1$, which is close to the wave length $19\delta_1$ of the most unstable two-dimensional disturbances considered previously.

For small values of the wave number β, there have been questions regarding both the accuracy of the preceding calculation methods and the validity of the approximate disturbance equations that are normally used in the investigation of the vortex instability of boundary layers. The first question was clarified by Hämmerlin[69], but the second is not completely settled as yet, although some tentative conclusions have been stated by Hämmerlin[70] and Di Prima and Dunn[71]. For wave numbers large enough to be of physical significance, it does not appear that the main results of the theory would be significantly altered by further refinements.

[1]The Blasius boundary layer (with $U_1'(x) = 0$) could occur on a curved plate in a non-uniform stream.

[2]Eq. (2) is altered to allow for variation of α with x, so that the exponential factor is $\exp\left[i\left(\int_0^x \alpha dx + \beta z - \omega t\right)\right]$ and $\alpha_r = \omega = 0$ for the class of disturbances considered.

EFFECTS OF COMPRESSIBILITY AND HEAT TRANSFER ON BOUNDARY LAYER STABILITY

The discussion up to this point has been limited to the stability of fluid motion in an incompressible fluid. In many problems of physical interest, compressibility and heat transfer phenomena are important, and these require an extended version of the theory. The fundamental paper of Lees and Lin[72] in 1946 was the first to carry out the generalization of the stability theory to the case of a compressible, viscous heat-conducting fluid, and has been the starting point for most of the later investigations of the stability of compressible flows. Most of the results that have been obtained concern boundary layers, although there are also some limited results for jets and mixing layers.

The mathematical formulation of the theory now starts with the general Navier-Stokes equations for a compressible, viscous heat-conducting fluid, which include an equation expressing the conservation of energy in addition to equations expressing the conservation of momentum and mass as in the incompressible case. Also, a thermodynamic equation of state and relations expressing the dependence of the coefficients of viscosity and thermal conductivity on the thermodynamic variables are specified for the particular fluid of interest. Stability investigations are usually restricted to a perfect gas. In a manner similar to that previously described for the incompressible boundary layer, linearized disturbance equations are derived for the disturbances (u_1, v_1, w_1, p_1, ρ_1, T_1) in the velocity components, pressure, density and temperature, respectively. Then, a consideration of elementary disturbances in the form of waves, essentially as described by Eq. (1) or (2), plus the assumption of a nearly parallel flow, lead to a system of ordinary differential equations for the corresponding disturbance amplitude functions (f, g, h, π, r, θ). When the equations are in a non-dimensional form, with the boundary layer thickness δ and the local free-stream values of the physical variables as reference quantities, the non-dimensional parameters include the local Mach number M at the edge of the boundary layer in addition to the Reynolds number R, and the coefficients in the equations depend on the local boundary layer temperature distribution $T(y)$ in addition to the velocity distribution $U(y)$. Thus, the Mach number and the wall temperature or heat-transfer rate, in addition to the parameters considered in the incompressible case, affect the stability characteristics, through their effects on $U(y)$ and $T(y)$. The system of equations, which is of order six for two-dimensional disturbances, cannot be conveniently reduced to a single equation analogous to the Orr-Sommerfeld Eq. (3), and is normally investigated as a system. The approximations inherent in these equations have been examined by Dunn and Lin[73, 74] and Cheng[75], with conclusions generally similar to those in

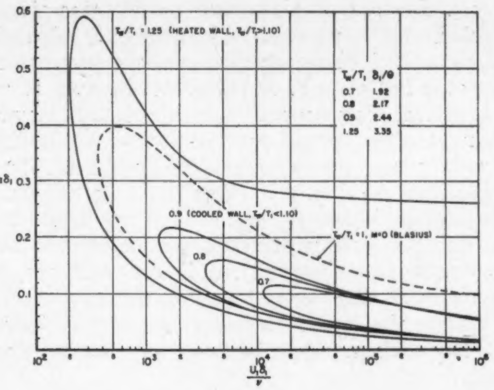


Figure 10

Curves of neutral stability for the boundary layer on a flat plate with heat transfer at a free-stream Mach number $M = 0.7$; U_1 = free-stream velocity, T_w/T_1 = ratio of wall to free-stream temperature, θ = momentum thickness, δ_1 = displacement thickness (after Lees[76])

the incompressible case. It appears, however, that the accuracy decreases at high Mach numbers.

Lees and Lin[72] considered two-dimensional disturbances (i.e., waves travelling in the main flow direction, with $\beta = 0$) and Dunn and Lin[73, 74] later investigated three-dimensional disturbances (i.e., oblique waves travelling at an angle to the main flow direction) as defined by Eq. (2). Lees and Lin reached several conclusions from their analytical investigation. One of the most interesting conclusions concerns the existence of the so-called supersonic disturbances, which have a supersonic phase velocity relative to the free-stream velocity ($c < 1 - (1/M)$ or $c > 1 + (1/M)$) and propagate outside the boundary layer with constant amplitude. Since there are no discrete eigenvalues for such disturbances, they are usually assumed to have no significance in the stability problem. Although they are probably relevant to the problem of sound radiation into or out of the boundary layer, their physical significance has not been completely clarified, and further study would be desirable. In addition to the supersonic disturbances, there are subsonic disturbances (with $1 > c > 1 - (1/M)$) whose amplitude diminishes to zero outside the boundary layer, which correspond to the instability waves of the incompressible case and lead to a similar eigenvalue problem.

For the calculation of the eigenvalues, asymptotic methods for large values of the Reynolds number are required, just as in the incompressible case. Lees and Lin[72] extended the asymptotic procedure previously developed for incompressible flow[21], and on this basis Lees[76] carried out comprehensive numerical calculations of the stability of the compressible boundary layer with zero free-stream pressure gradient and various ratios of wall temperature to free-stream temperature. For an insulated wall, the results show that the effect of Mach number on the minimum critical Reynolds number is not large up to $M = 1.3$. However, the effect of cooling or heating (i.e., wall temperature less than or greater than the adiabatic wall temperature) is very pronounced, as shown by the curves of neutral stability in Figure 10. For a supersonic free stream ($M > 1$), Lees showed that in many cases the boundary layer can even be completely stabilized with respect to two-dimensional disturbances by sufficient cooling at the wall. More elaborate calculations of the critical wall temperatures for complete stabilization were made in later investigations, the most recent being the calculations of Dunn and Lin[74], illustrated in Figure 11. Other investigations following those of Lees and Lin also studied the influence of pressure gradient and fluid suction or injection, with results similar to those in the incompressible case. However, fluid injection of a cool fluid can have a resultant stabilizing effect, if the stabilizing effect of the boundary layer cooling is large enough to counteract the basically destabilizing effect of injection. Some recent work in this direction is described in Reference 77. Lin's book[8] contains a summary of various special applications of the Lees-Lin stability theory up to 1954.

The main limitations of the theory of Lees and Lin are the restriction to two-dimensional disturbances mentioned previously, and also a restriction to small wave speeds imposed by the asymptotic method used, which in practice amounts to a restriction on the free-stream Mach number to values not too much larger than one. Thus, calculations based on the Lees-Lin theory become inaccurate at supersonic Mach numbers[k].

The investigations of Dunn and Lin[73, 74] were undertaken in order to remove these limitations. The conclusions for three-dimensional effects are developed in detail in Reference 74 and a summary of a new asymptotic method suitable for high Mach numbers is also given. Squire's result for the incompressible case, that three-dimensional disturbances are more stable than two-dimensional ones, is not in general valid for the compressible case. However, the simplified equations for three-dimensional disturbances, which result when the parallel flow approximations are consistently applied, transform to a system of the

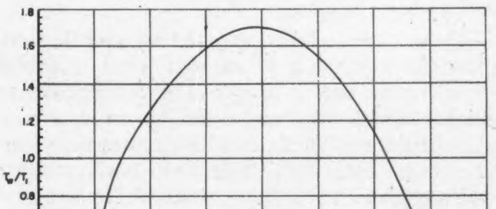


Figure 11

Critical temperature ratios for complete stability with respect to two-dimensional disturbances of the boundary layer on a cooled flat plate (after Dunn and Lin[74])

[k]Recent calculations[77, 79] suggest that in many cases the Lees-Lin theory is accurate up to about $M = 2$.

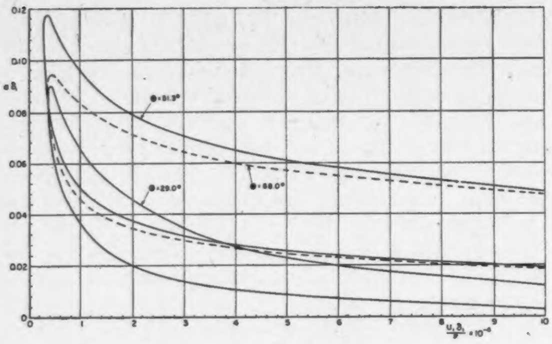


Figure 12

Curves of neutral stability for oblique waves in the boundary layer on a cooled flat plate at a free-stream Mach number $M = 1.6$ and a ratio of wall to free-stream temperature $T_w/T_1 = 1.073$; U_1 = free-stream velocity, θ = momentum thickness, δ_1 = displacement thickness $= 3.25\theta$, $\Theta = \arctan(\beta/\alpha)$ = direction angle of wave normal (after Dunn and Lin[74])

same mathematical form as that for two-dimensional disturbances, so that the investigation of the general case is not significantly more difficult. An additional parameter is involved in the three-dimensional problem, the direction angle arctan (β/α) of the wave-normal. In contrast to the situation for two-dimensional disturbances, the boundary layer can never be completely stabilized with respect to three-dimensional disturbances. Figure 12 shows neutral curves for various values of the direction angle for $M = 1.6$ and $T_w/T_1 = 1.073$, a case for which two-dimensional disturbances are completely stable (Figure 11). Although unstable three-dimensional disturbances are clearly present in the usual Reynolds number range, it turns out that only a small amount of additional cooling (i.e., a slightly smaller value of T_w/T_1) is required to increase the minimum critical Reynolds number for all unstable three-dimensional disturbances to a value far beyond the range of practical significance. Thus, cooling is still an effective means of stabilizing the boundary layer in a gas.

The usefulness of the improved asymptotic method outlined in Reference 74 at high Mach numbers is not completely settled as yet. The numerical results of Reference 74 were calculated by an approximate version of the new method, which represents a partial improvement over the earlier Lees-Lin method but would not give results much different from the latter in the cases considered. Some exploratory calculations based on the complete eigenvalue relation of Reference 74, which were carried out independently by Dunn and Mack in 1956, led to unrealistic answers. The difficulties were traced to an unnecessary approximation in the final derivation of the eigenvalue relation from the asymptotic solutions of the disturbance equations, which had originally been made for the sake of "consistency" in the approximation scheme, and an analytical error in one of the asymptotic solutions derived in Reference 73. A complete and revised version of the improved theory is presented in an internal NRC report[78], which has not been published as yet. The revised eigenvalue relation was given to Mack in 1956[l], who recently completed calculations of the neutral curve for the boundary layer over an insulated flat plate at $M = 2.2$ in the case of two-dimensional disturbances[79], with the results shown in Figure 13. The neutral curves for $M = 0$ (the Blasius boundary layer in incompressible flow) and $M = 1.3$ (calculated previously by Lees[76]) are also shown for comparison. Although the minimum critical Reynolds number decreases with Mach number in this range, the change is not large. There is some evidence, both experimental and theoretical, that the trend reverses at a somewhat higher Mach number, so that the critical Reynolds number increases in the hypersonic range.

Reshotko[38] recently calculated a neutral curve at $M = 2.2$, starting from a different theoretical formulation in which some of the terms previously neglected in the disturbance equations by the parallel flow approximations are retained[m]. This neutral curve almost coincides with Mack's curve on the top branch, but is about 20% higher on the lower branch. Another neutral curve at $M = 2.2$ calculated by Mack using the Lees-Lin method is about as close to each of the other two. It seems to the present author that the numerical discrepancies between the various calculations of the neutral curve at $M = 2.2$ are of the order of magnitude of the errors to be expected from the asymptotic approximations inherent in any of the schemes used. Reshotko also made a few calculations at $M = 5.6$ based on the three different methods, obtaining results quite different from one another numerically and difficult to interpret.

The experimental neutral curve of Laufer and Vrebalovich[80] for the case $M = 2.2$ agrees closely with the theoretical one on the upper branch, but is over 100% higher on the lower branch. Demetriades' ex-

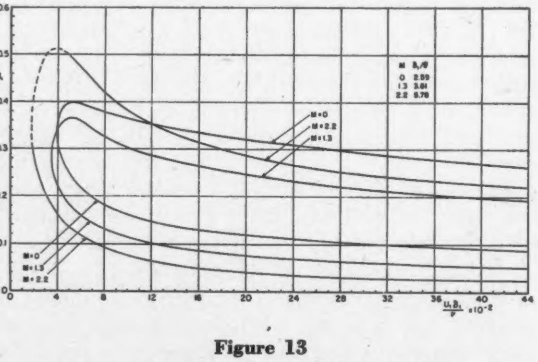


Figure 13

Curves of neutral stability for the boundary layer on an insulated flat plate at free-stream Mach numbers $M = 0$, 1.3 and 2.2; U_1 = free-stream velocity, δ_1 = displacement thickness, θ = momentum thickness (after Lees[76] and Mack[79])

[l]The "corrected Dunn-Lin eigenvalue relation" of References 38 and 79 is identical to the eigenvalue relation of Reference 78 in the special case of an insulated wall, but not otherwise. Reshotko[38], who made an independent derivation, apparently overlooked the analytical error referred to above.

[m]There are still several terms neglected that are formally of the same order of magnitude as these new terms.

perimental neutral curve for the case $M = 5.6$[81] is located at values of αR much larger than those at some of the points calculated by Reshotko[38]. The reasons for the discrepancies between theory and experiment are not clear at this time. Lees and Reshotko[38, 82] suggest that the asymptotic methods for large αR traditionally used in the stability theory are not adequate at high Mach numbers. This would be so if the values of αR were not large enough, but the experimental values of αR for neutral stability do not seem especially small[n]. In the author's opinion, the evidence at this stage is inconclusive. However, the recent concentration of research effort on this problem seems likely to clarify the situation in the near future.

In comparison with the work on boundary layers, theoretical research on the stability of other compressible laminar flows has been rather limited. The main investigations are those for jet stability in the inviscid limit, summarized in Pai's book[83], and a similar study for the compressible mixing layer by Lin[84]. An interesting result is that the mixing layer is likely to be stable when the relative speed of the two parallel streams exceeds the sum of their velocities of sound. A recent investigation by Miles[85] may have some relevance to this problem.

Although the problem of heat transfer effects on the stability of laminar flows of a gas has attracted much more attention, the same problem in the case of a liquid also has interesting features. Theoretical investigations are somewhat simpler for a liquid than for a gas, since the density variations can usually be neglected, but the temperature dependence of the viscosity and thermal conductivity must be considered when temperature changes are large. Since the viscosity of a liquid changes with temperature in the opposite sense to that for a gas, the effects of heat transfer are opposite to those for a gas. For example, McIntosh[86] considered the two-dimensional stability of the boundary layer flow of water over a flat plate at a temperature 50°C above or below the free-stream temperature, and found that heating increased the minimum critical Reynolds number while cooling decreased it from the value for an adiabatic wall ($R_{1c} = 420$, see Figure 4) by factors of the order of ten[o]. The sensitivity of this type of instability to heating or cooling at the wall is evidently just as great for water as for air. On the other hand, Di Prima and Dunn[71] considered the boundary layer flow of water over a curved surface, and showed that under exactly the same wall temperature conditions the three-dimensional stability characteristics changed relatively little from those for an adiabatic wall, as in Figure 9. The effect of heat transfer on this vortex instability is just as small as the effect Görtler[67] found for the free-stream pressure gradient, at least in the case of water. A similar invesigation has not as yet been made in the case of a gas.

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[n]Even with R based on physical properties evaluated at a suitably defined reference temperature.

[o]This investigation was not published, but the results are quoted in Reference 71.

ENGINEERING SIGNIFICANCE OF THE STABILITY THEORY

Although fundamental research has been carried out for many years on the problem of instability of laminar flows and transition to turbulence, the influence of this on practical engineering developments has been relatively small until recently. In fact, as pointed out previously, many of the theoretical results were not widely recognized outside of Germany until after 1945. However, the rapid technological advances of recent years, particularly in aeronautics and missile engineering, have led to critical design problems, which require for their solution the most advanced knowledge available. For example, since the laminar skin friction and heat transfer coefficients are usually much smaller than the turbulent ones, there has been considerable interest in the possibility of reducing frictional drag and heat transfer in the case of an aircraft or missile in flight, by maintaining laminar boundary layers over larger surface areas by means of stabilization techniques suggested by the stability theory. It should, of course, be emphasized that the theory is not yet capable of predicting the exact location of transition to turbulent flow, which usually occurs at a considerably higher Reynolds number than the theoretical minimum critical Reynolds number at which the laminar flow becomes unstable with respect to small disturbances. A complete theory of transition would have to deal with non-linear and other effects not considered in the present form of the stability theory. However, the factors affecting the stability of a laminar flow have been found by experience to be important factors in the transition process in many cases. For example, in the case of a boundary layer subjected to small enough extraneous disturbances, any factor that increases the minimum critical Reynolds number, such as a favourable pressure gradient, also increases the transition Reynolds number.

The simplest practical application of a stabilization technique is in the design of the laminar flow aerofoil, which was developed in the late 1930's. On an aerofoil of this type, the point of maximum thickness is located farther back towards the trailing edge than usual, so that the position of the pressure minimum is also farther back. Thus, a favourable pressure gradient (i.e., a decreasing pressure in the direction of flow) extends over a larger part of the aerofoil, maintaining a laminar boundary layer in this region. The resultant drag coefficient of the aerofoil is less than if the turbulent boundary layer, with its higher skin friction coefficient, extended farther towards the leading edge. Various suction techniques for stabilizing the laminar boundary layer have also been investigated, and although they have not been used much in practice as yet, they appear to have a wide range of usefulness. Schlichting[4, 9] has surveyed the practical applications of stabilization techniques and other aspects of boundary layer control in low speed flight from the early German work to the present. Recent developments are discussed by Lachmann[87] and Schmued[88], and a comprehensive treatment of the whole subject of boundary layer control is given in a new book edited by Lachmann[89].

In high speed flight, other boundary layer characteristics in addition to the skin friction become of importance. In particular, the heat transfer to missiles at hypersonic speeds leads to difficult design problems, since operating conditions are often such that laminar, transitional and turbulent boundary layers are of equal importance. The factors affecting the location of transition, as well as the possibility of reducing the heat transfer by maintaining completely laminar boundary layers, are therefore of great interest. However, the stability theory for compressible boundary layers does not appear to be advanced far enough yet to be a completely reliable guide in itself. The highly stable laminar boundary layers predicted by the theory for the case of a cooled wall can be realized under carefully controlled experimental situations (see Reference 12, pages 63 to 67), but are not always attained in practice. Important factors not yet considered in the theory, particularly roughness of the surface, are often dominant in such situations and lead to relatively low transition Reynolds numbers. For the present, the main reliance has to be placed on experimental research. However, the concepts and results of the stability theory are often found to be useful in designing experiments and interpreting experimental results. The experimental research on transition up to 1958 has been surveyed by Dryden[13] and Morkovin[14], and some of the more recent work on boundary layer transition in high speed flow is described in References 90 to 92. In the case of supersonic aircraft, the interior cabin noise due to a turbulent boundary layer over the exterior surface is another problem in addition to heat transfer, and provides further motivation for maintaining laminar boundary layers.

Some other aspects of the stability theory of laminar flows have not attracted as much attention, but appear to have considerable practical significance. For example, the highly stable character of the laminar mixing layer under certain conditions at high Mach numbers suggested by theory[84] has been verified experimentally[93]. At low speeds, the mixing layer is highly unstable, and as a result is completely turbulent in most practical situations. At high speeds, the mixing layer can often have a long extent of laminar flow before transition to turbulence. Mixing layers are of common occurrence in flow problems, and their transition characteristics are important, since they sometimes influence the character of a large part of the flow field external to the mixing layer through their effect on the mixing layer properties.

The possibility of drag reduction in the case of water craft by maintaining laminar boundary layers does not seem to have been investigated as thoroughly as in the case of aircraft. Since the major part of the drag of an underwater craft can be skin-friction drag, this possibility appears especially attractive. In addition to the use of a favourable pressure gradient or suction, heating the surface would appear from the theoretical results[71, 86] to be another convenient stabilization method. Although experimental investigation by Siegel and Shapiro[94] on the boundary layer at the entrance to a pipe revealed no significant change in transition Reynolds number due to heating, this result may have been due to a relatively large free-stream turbulence level. Under natural conditions, when an object moves in a large body of fluid at rest, the free-stream turbulence is usually small, and large increases in the transition Reynolds number might be realized. In addition to drag reduction, stabilization would also reduce the hydrodynamic sound (part of which is produced by the turbulent boundary layer) by reducing the extent of the turbulent boundary layer. This result might also be of importance in some situations.

Another method of stabilizing the boundary layer flow around an object has been suggested recently by Kramer[95] that involves the use of a surface coating of elastic damping material. Since previous results of the stability theory indicate that under given external flow conditions the most unstable disturbances involve only a limited range of frequencies, the physical properties of the surface material can be adjusted so that these frequencies are damped most effectively. Kramer's experimental results[95] seem to show that this procedure does in fact stabilize the boundary layer. A recent theoretical investigation by Benjamin[96] suggests that stabilization can actually be achieved in two different ways, one of which corresponds to Kramer's experiment. It has been suggested that stabilization by distributed damping might be the means by which the porpoise (a marine animal) is apparently able to keep its boundary layer laminar[p] and hence its drag low enough to explain the fast swimming speeds it has been observed to achieve.

SOME ASPECTS OF THE COMPLETE TRANSITION PROBLEM

The fundamental deficiencies in the stability theory that prevent it from being a complete theory of transition are the omission of nonlinear effects in most theoretical investigations and the absence of any consideration of free-stream disturbances. The significance of nonlinearity has been widely recognized for many years, but the importance of free-stream disturbances has not been sufficiently emphasized until recently (e.g., see References 12 and 14). In the case of the boundary layer, for example, it is reasonable to expect that the transition Reynolds number could be arbitrarily large compared with the minimum critical Reynolds number if the magnitude of the free-stream disturbances were small enough[q]. On the other hand, when the free-stream disturbances become large, the available evidence indicates that the transition Reynolds number approaches the critical Reynolds number and becomes relatively insensitive to the factors that normally have a large effect on stability. Evidently stability considerations alone are not sufficient to determine the transition location (except,

[p]The stabilizing influence of heat transfer might also be a contributing factor. The porpoise, which is a mammal, would normally have a surface temperature somewhat warmer than the surrounding water temperature.

[q]Although a trend in this direction has been observed in the case of pipe flow with small inlet-flow disturbances, the experimental evidence in this case of boundary layers and other laminar flows is inconclusive.

perhaps, a lower limit to it) and any complete theory of transition would have to include the free-stream disturbances. Probably their main role would be to supply some of the boundary conditions and/or initial conditions in a suitably defined initial value – boundary value problem for the boundary layer disturbances. From this point of view, randomness of the boundary layer disturbances is just a consequence of randomness of the free-stream disturbances. Nonlinear effects are, of course, always important in the final stages of transition, and possibly in the initial stages as well if the free-stream disturbances are large enough.

In the traditional formulation of the stability problem, an arbitrary disturbance is regarded as a superposition of elementary disturbances (or normal modes) satisfying all of the spatial boundary conditions, and the problem reduces to an initial value problem with the time as independent variable. Very little work has been done on the full problem beyond Haupt's proof of the superposition theorem for plane Couette flow subjected to an arbitrary two-dimensional initial disturbance[97]. A few later investigations have examined the higher modes of disturbance for other flows, the most recent being an elaborate study by Grohne[98]. In contrast to the approach using normal modes, Case[99] has recently applied the more direct Laplace-transform technique to the initial value problem for plane Couette flow in the inviscid limit. It is the author's opinion that, although the initial value problem of the traditional formulation may correspond to the physical transition process in some cases, it may not always be relevant. In the case of the boundary layer, for example, transition is observed to take place as a result of disturbance growth with distance, rather than time[r], and it would seem that the initial value problem should be formulated accordingly. Such a formulation would correspond to the elementary disturbances described at the end of the section dealing with "Stability of Parallel Flows". It should be emphasized, of course, that there are many conflicting (or apparently conflicting) views on the detailed nature of the transition process at the present time, and a universally accepted definition of the mathematical and physical problem is not available. A complete solution to the transition problem will probably not be obtained for many years.

CONCLUSION

As promised in the introduction, this survey has been limited to topics of most interest in engineering, and as a result may give an unbalanced view of the work that has been done on the stability of fluid motion. There are many problems in astrophysics and geophysics, for which there is a large and rapidly growing research literature that can only be mentioned here. These include problems of the stability of the interface between two different fluids, and stability problems in which various combinations of such factors as heat transfer, electromagnetic effects, gravitation, Coriolis forces and surface tension are considered. Some of these problems are, of course, also of some interest in certain branches of engineering. The interested reader is referred to a brief treatment in Chapter 7 of Lin's book[8], and more comprehensive accounts by Chandrasekhar[40, 48]. A bibliography on stability problems in magneto fluid dynamics is included in Reference 100.

Even within the range of problems considered, research on the instability and transition of incompressible laminar flows, both theoretical and experimental, has been more extensive than indicated. The stability of three-dimensional boundary layers has been investigated in a fundamental paper by Gregory, Stuart, and Walker[101], and more extensive calculations were later made by Brown[102]. The destabilizing effect of large scale surface irregularities (waves) on boundary layer stability has been studied by Pretsch[49] and Spence and Randall[103]. New asymptotic solutions of the Orr-Sommerfeld equation have been developed recently by Lin and Rabenstein[104], which should provide a more adequate analytical basis for stability calculations (see References 7 and 8). Another approach to the laborious numerical solution of the stability problem is by means of direct numerical integration of the Orr-Sommerfeld equation. Because of the progress in mathematical understanding of solutions of the Orr-Sommerfeld equation and the development of modern high speed computing machines, this approach has become more feasible in recent years. Promising results have been obtained by Thomas[105] (Figure 2) and Brown[102]. Important theoretical advances have also been made recently on several aspects of the transition problem beyond those treated in the linear theory. For example, there are the investigations of nonlinear effects by Stuart[24], Benney and Lin[106], and the recent German work on nonlinear effects and secondary instability, summarized by Görtler[11]. In addition to the theoretical research, fundamental experimental investigations of the detailed mechanism of instability and transition have continued steadily through the years. An example of recent work is the investigation of Klebanoff and Tidstrom[107].

From the practical point of view, the stability theory has provided ideas that are already proving useful in engineering problems and that will likely prove even more useful in the future. However, the theory must be regarded as being rather limited in direct applicability as yet, since there are many aspects of practical problems that are beyond its present scope. Great care is required in going from the highly idealized theoretical problems to the much more complicated practical ones. Two of the most important limitations are the lack of any detailed theoretical treatment of the effect of surface roughness (small scale irregularities) and the limited number of stability investigations of three-dimensional basic flows. The theory for compressible flows is especially undeveloped, since the solutions of many stability problems in the incompressible case have not as yet been extended to the compressible case. Important examples are the stability of the compressible mixing layer (which has been examined only in the inviscid limit)

[r]Unless one adopts the rather artificial device of "following the disturbance" as it travels downstream. The difficulty is that a mathematical disturbance is not easily defined physically. It does not follow the fluid motion, for example.

and the stability of compressible boundary layers over curved surfaces with respect to three-dimensional disturbances of the vortex type (Görtler vortices). For the compressible case, three-dimensional disturbances can also be important in many stability problems not involving curved boundaries, and they cannot be dismissed as readily as in the incompressible case. Extensions of the stability theory in these various directions would appear to be relatively straightforward, except perhaps in the case of surface roughness, and should lead to a better understanding of many of the factors affecting instability and transition in practice.

Although the stability theory can only supply part of the answer to the complete problem of transition to turbulence, the practical significance of its contribution should not be underrated. If extraneous disturbances are small enough, then the minimum critical Reynolds number predicted by the theory is a valid lower limit to the transition Reynolds number, and the theoretical predictions regarding the influence of various factors on stability are reliable. Such conclusions are often of great value in engineering situations. What disturbances are important and how small they have to be are questions that must be answered by experimental investigations with the aid of the available theoretical knowledge. In this respect, further developments of the theory will evidently be helpful.

REFERENCES

(1) Reynolds, O. – *An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels,* AND ALSO *On the Dynamical Theory of Incompressible Viscous Fluids and the Determination of the Criterion,* PHIL. TRANS. ROY. SOC. LONDON, SERIES A, VOL. 174, P 935-982, 1883, AND VOL. 186, P 123-164, 1895. (REPRINTED IN SCIENTIFIC PAPERS, VOL. 2, P 51-105, P 535-577; CAMBRIDGE UNIVERSITY PRESS, 3 VOLS., 1900-1903.)

(2) Lord Rayleigh – *On the Stability or Instability of Certain Fluid Motions,* PROC. LONDON MATH. SOC., VOL. 11, P 57-70, 1880; VOL. 19, P 67-74, 1887; VOL. 27, P 5-12, 1895. (REPRINTED IN SCIENTIFIC PAPERS, VOL. 1, P 474-487; III, P 17-23; IV, P 203-209; CAMBRIDGE UNIVERSITY PRESS, 6 VOLS., 1899-1920.) SEE ALSO SCIENTIFIC PAPERS III, P 575-584; IV, P 197-204, P 266-275, P 341-349, P 447-453.

(3) Lin, C. C. – *The Theory of Hydrodynamic Stability,* CAMBRIDGE UNIVERSITY PRESS, 1955.

(4) Schlichting, H. – *Boundary Layer Theory, 4th Edition,* MCGRAW HILL, NEW YORK, 1960.

(5) Schlichting, H. – *Entstehung der Turbulenz* (ENCYCLOPEDIA OF PHYSICS), EDITED BY S. FLÜGGE, VOL. VIII/1, FLUID DYNAMICS I, P 351-450, SPRINGER-VERLAG, BERLIN-GÖTTINGEN-HEIDELBERG, 1959.

(6) Lin, C. C. – *Stability of Laminar Flows,* APPLIED MECHANICS REVIEWS, VOL. 10, P 1-3, JANUARY, 1957.

(7) Lin, C. C. – *On the Stability of the Laminar Boundary Layer,* SYMPOSIUM ON NAVAL HYDRODYNAMICS, WASHINGTON, D.C., SEPTEMBER, 1956; PUBLICATION 515, P 353-373, NATIONAL ACADEMY OF SCIENCES – NATIONAL RESEARCH COUNCIL, WASHINGTON, 1957.

(8) Lin, C. C. – *On the Instability of Laminar Flow and its Transition to Turbulence,* BOUNDARY LAYER RESEARCH, EDITED BY H. GÖRTLER, P 144-157, SPRINGER-VERLAG, BERLIN-GÖTTINGEN-HEIDELBERG, 1958.

(9) Schlichting, H. – *Some Developments in Boundary Layer Research in th Past Thirty Years,* J. ROY. AERO. SOC., VOL. 64, P 64-79, FEBRUARY, 1960.

(10) Serrin, J. – *Mathematical Principles of Classical Fluid Mechanics,* (ENCYCLOPEDIA OF PHYSICS), EDITED BY S. FLÜGGE, VOL. VIII/1, FLUID DYNAMICS I, P 253-260, SPRINGER-VERLAG, BERLIN-GÖTTINGEN-HEIDELBERG, 1959.

(11) Görtler, H. – *Einige Neuere Ergebnisse zur Hydrodynamischen Stabilitätstheorie,* Z. FLUGWISS, VOL. 8, P 1-8, JANUARY, 1960.

(12) Dryden, H. L. – *Transition From Laminar to Turbulent Flow,* TURBULENT FLOWS AND HEAT TRANSFER, EDITED BY C. C. LIN, SECTION A, P 3-74, PRINCETON UNIVERSITY PRESS, 1959.

(13) Kuethe, A. M. – *Some Features of Boundary Layers and Transition to Turbulent Flow,* J. AERO. SC., VOL. 23, P 444-452, MAY, 1956.

(14) Morkovin, M. V. – *Transition From Laminar to Turbulent Shear Flow – A Review of Some Recent Advances in Its Understanding,* TRANS. ASME, VOL. 80, P 1121-1128, JULY, 1958.

(15) Sommerfeld, A. – *Ein Beitrag zur Hydrodynamischen Erklärung der Turbulenten Flüssigkeitsbewegung,* PROC. 4TH INT. CONGR. MATH., P 116-124, ROME, 1908.

(16) Heisenberg, W. – *Uber Stabilität und Turbulenz von Flüssigkeitsströmen,* ANN. D. PHYSIK, VOL. 74, P 577-627, 1924. ENGLISH TRANSLATION, NACA TECH. MEMO 1291, JUNE, 1951.

(17) Taylor, G. I. – *Stability of a Viscous Liquid Contained Between Two Rotating Cylinders,* PHIL. TRANS. ROY. SOC. LONDON SERIES A, VOL. 223, P 289-343, 1923.

(18) Tollmien, W. – *Uber die Entstehung der Turbulenz,* NACHR. GES. WISS. GÖTTINGEN, MATH.-PHYS. KLASSE P 21-44, 1929. ENGLISH TRANSLATION, NACA TECH. MEMO. 609, 1931.

(19) Schlichting, H. – *Zur Entstehung der Turbulenz bei der Plattenströmung,* NACHR. GES. WISS. GÖTTINGEN, MATH.-PHYS. KLASSE, P 181-208, 1933.

(20) Schlichting, H. – *Amplitudenverteilung und Energiebilanz der Kleinen Störungen bei der Plattengrenzschicht,* NACHR. GES. WISS. GÖTTINGEN, MATH.-PHYS. KLASSE, VOL. 1, P 47-78, 1935. ENGLISH TRANSLATION, NACA TECH. MEMO. 1265, APRIL, 1950.

(21) Lin, C. C. – *On the Stability of Two-Dimensional Parallel Flows. Parts I, II, III,* QUART. APPL. MATH., VOL. 3, P 117-142, P 218-234, P 277-301, 1945.

(22) Schubauer, G. B., and Skramstad, H. K. – *Laminar Boundary Layer Oscillations and Transition on a Flat Plate,* NACA REP. NO. 909, 1948. ORIGINALLY ISSUED AS NACA ACR, APRIL, 1943. SEE ALSO, J. AERO. SCI., VOL. 14, P 69-78, FEBRUARY, 1947; J. RES. NAT. BUR. STAND., VOL. 38, P 251-292, FEBRUARY, 1947.

(23) Stuart, J. T. – *On the Effects of the Reynolds Stress on Hydrodynamic Stability,* Z. ANGEW. MATH. MECH., SONDERHEFT, P S32-S38, 1956.

(24) Stuart, J. T. – *On the Non-Linear Mechanics of Hydrodynamic Stability,* J. FLUID MECH., VOL. 4, P 1-21, MAY, 1958.

(25) Dunn, D. W. – MATH. REVIEWS, VOL. 18, P 252-253, MARCH, 1957.

(26) Squire, H. B. – *On the Stability of the Three-Dimensional Disturbances of Viscous Flow Between Parallel Walls,* PROC. ROY. SOC. LONDON, SERIES A, VOL. 142, P 621-628, 1933.

(27) Orr, W. McF. – *The Stability or Instability of the Steady Motions of a Liquid,* PROC. ROY. IRISH ACAD., SERIES A, VOL. 27, P 9-27, P 69-138, 1906-1907.

(28) Tollmien, W. – *Ein Allgemeines Kriterium der Instabilität Laminarer Geschwindigkeitsverteilungen,* NACHR. GES. WISS. GÖTTINGEN, MATH.-PHYS. KLASSE, VOL. 50, P 79-114, 1935. ENGLISH TRANSLATION NACA TECH. MEMO. 792, 1936.

(29) Wasow, W. – *On Small Disturbances of Plane Couette Flow*, J. Res. Nat. Bur. Stand., Vol. 51, p 195-202, October, 1953.

(30) Schlichting, H. – *Uber die Stabilität der Couette-strömung*, Ann. d. Physik, Vol. 14, p 905-936, 1932.

(31) Shen, S. F. – *Calculated Amplified Oscillations in the Plane Poiseuille and Blasius Flows*, J. Aero. Sci., Vol. 21, p 62-64, January, 1954.

(32) Sexl, T., and Spielberg, K. – *Zum Stabilitätsproblem der Poiseuille-Strömung*, Acta Phys. Austriaca, Vol. 12, p 9-28, 1958.

(33) Corcos, G. M., and Sellars, J. R. – *On the Stability of Fully Developed Flow in a Pipe*, J. Fluid Mech., Vol. 5, p 97-113, January, 1959.

(34) Spielberg, K., and Timan, H. – *On Three- and Two-Dimensional Disturbances of Pipe Flow*, J. Appl. Mech., Vol. 27E, p 381-389, September, 1960.

(35) Tatsumi, T. – *Stability of the Laminar Inlet-Flow Prior to the Formation of the Poiseuille Regime. Parts I, and II*, J. Phys. Soc. Japan, Vol. 7, p 489-495, p 495-502, 1952.

(36) Leite, R. J. – *An Experimental Investigation of the Stability of Poiseuille Flow*, J. Fluid Mech., Vol. 5, p 81-97, January, 1959.

(37) Battin, R. H., and Lin, C. C. – *On the Stability of the Boundary Layer Over a Cone*, J. Aero. Sci., Vol. 17, p 453, July, 1950.

(38) Reshotko, E. – *Stability of the Compressible Laminar Boundary Layer*, Cal. Inst. Tech., GALCIT Memo. No. 52, January, 1960.

(39) Battin, R. H. – Ph.D. thesis, Dept. of Math., Mass. Inst. Tech., 1951.

(40) Chandrasekhar, S. – *On Characteristic Value Problems in High Order Differential Equations Which Arise in Studies on Hydrodynamic and Hydromagnetic Stability*, American Math. Monthly, Vol. LXI, p 32-45, August-September, 1954.

(41) Chandrasekhar, S., and Reid, W. H. – *On the Expansion of Functions Which Satisfy Four Boundary Conditions*, Proc. Nat. Acad. Sci. USA, Vol. 43, p 521-527, June, 1957.

(42) Chandrasekhar, S. – *The Stability of Viscous Flow Between Rotating Cylinders*, Proc. Roy. Soc. London, Series A, Vol. 246, p 301-311, August, 1958.

(43) Di Prima, R. C. – *The Stability of Viscous Flow Between Rotating Concentric Cylinders with a Pressure Gradient Acting Round the Cylinders*, J. Fluid Mech., Vol. 6, p 462-468, October, 1959.

(44) Dean, W. R. – *Fluid Motion in a Curved Channel*, Proc. Roy. Soc. London, Series A, Vol. 121, p 402-420, 1928.

(45) Yih, C. S., and Sangster, W. M. – *Stability of Laminar Flow in Curved Channels*, Phil. Mag. (8), Vol. 2, p 305-310, March, 1957.

(46) Hämmerlin, G. – *Die Stabilität der Strömung in Einem Gekrümmten Kanal*, Archive Rat. Mech. Anal., Vol. 1, p 212-224, May, 1958.

(47) Reid, W. H. – *On the Stability of Viscous Flow in a Curved Channel*, Proc. Roy. Soc. London, Series A, Vol. 244, p 186-198, March, 1958.

(48) Chandrasekhar, S. – *Hydrodynamic and Hydromagnetic Stability*. Oxford University Press (to be published).

(49) Pretsch, J. – *Die Stabilität Einer Ebenen Laminarströmung bei Druckgefälle und Druckanstieg*, Jahrbuch der deutschen Luftfahrtforschung, I 58-75, 1941. English translation, Great Britain, MAP-VG52, Reports and Translations No. 22.

(50) Schlichting, H., and Ulrich, A. – *Uber die Berechnung der Umschlagstelle Laminar-Turbulent*, Jahrbuch der deutschen Luftfahrtforschung, I 8-35, 1952. (Complete text in Bericht S 10 of the Lilienthal-Gesellschaft für Luftfahrtforschung.)

(51) Zaat, J. A. – *Numerische Beiträge zur Stabilitätstheorie der Grenzschichten*, Boundary Layer Research, edited by H. Görtler, p 127-138, Springer-Verlag, Berlin-Göttingen-Heidelberg, 1958.

(52) Ulrich, A. – *Theoretische Untersuchungen über die Widerstandsersparnis durch Laminarhaltung mit Absaugung*, Schriften d. dt. Akad. d. Luftfahrtforschung, Vol. 8B, No. 2, 1944. English translation, NACA Tech. Memo. 1121, June, 1947.

(53) Hollingdale, S. H. – *Stability and Configuration of the Wakes Produced by Solid Bodies Moving Through Fluids*, Phil. Mag., (7), Vol. 29, p 209-257, 1940.

(54) Savic, P. – *On Acoustically Effective Vortex Motion in Gaseous Jets*, Phil. Mag., (7), Vol. 32, p 245-252, 1941. See also Vol. 34, p 139-144, 1943.

(55) Lessen, M. – *On the Stability of the Free Laminar Boundary Layer Between Parallel Streams*, NACA Rep. No. 979, 1950. Previously issued as Tech. Note 1929, 1949.

(56) Tatsumi, T., and Kakutani, T. – *The Stability of a Two-Dimensional Laminar Jet*, J. Fluid Mech., Vol. 4, p 261-275, July, 1958.

(57) Howard, L. N. – *Hydrodynamic Stability of a Jet*, J. Math. Phys., Vol. 37, p 283-298, January, 1959.

(58) Curle, N. – *Hydrodynamic Stability of Laminar Wakes*, Physics of Fluids, Vol. 1, p 159-160, 1958.

(59) Lin, C. C. – Math. Reviews, Vol. 19, p 202, 1958.

(60) Curle, N. – *On Hydrodynamic Stability in Unlimited Fields of Viscous Flow*, Proc. Roy. Soc. London, Series A, Vol. 238, p 489-501, January, 1957.

(61) Arkhipov, V. N. – *The Formation of Streaming Fluctuations Behind a Solid Obstacle*, Soviet Physics-Doklady, Vol. 3, p 1117-1120, November-December, 1958.

(62) Tatsumi, T., and Gotoh, K. – *The Stability of Free Boundary Layers Between Two Uniform Streams*, J. Fluid Mech., Vol. 7, p 433-441, March, 1960.

(63) Lew, H. G. – *On the Stability of the Axially Symmetric Laminar Jet*, Quart. Appl. Math., Vol. 13, p 310-314, October, 1955.

(64) Kawaguti, M. – *The Critical Reynolds Number for the Flow Past a Sphere*, J. Phys. Soc. Japan, Vol. 10, p 694-699, 1955.

(65) Kawaguti, M. – *The Flow of a Viscous Fluid Past a Bluff Body*, Proceedings 9th Int. Congress Appl. Mech., Vol. 3, p 232-240, Brussels, 1957.

(66) Görtler, H. – *Uber den Einflusz der Wandkrummung auf die Entstehung der Turbulenz*, Z. angew. Math. Mech., Vol. 20, p 138-147, 1940.

(67) Görtler, H. – *Uber eine Dreidimensionale Instabilität Laminarer Grenzschichten an Konkaven Wänden*, Nachr. Ges. Wiss. Göttingen, Vol. 2, p 1-26, 1940. See also Z. Angew. Math. Mech., Vol. 21, p 250-252, 1941. English translation, NACA Tech. Memo. 1375, June, 1954.

(68) Smith, A. M. O. – *On the Growth of Taylor-Görtler Vortices Along Highly Concave Walls*, Quart. Appl. Math., Vol. 13, p 233-262, October, 1955.

(69) Hämmerlin, G. – *Uber das Eigenwertproblem der Dreidimensionalen Instabilität Laminarer Grenzschichten an Konkaven Wänden*, J. Rat. Mech. Analysis, Vol. 4, p 279-321, 1955.

(70) Hämmerlin, G. – *Zür Theorie der Dreidimensionalen Instabilität Laminarer Grenzschichten*, Z. Angew. Math. Phys., Vol. 7, p 156-164, 1956.

(71) Di Prima, R. C., and Dunn, D. W. – *The Effect of Heating and Cooling on the Stability of the Boundary Layer Flow of a Liquid Over a Curved Surface*, J. Aero. Sci., Vol. 23, p 913-916, October, 1956.

(72) Lees, L., and Lin, C. C. – *Investigation of the Stability of the Laminar Boundary Layer in a Compressible Fluid*, NACA Tech. Note 1115, September, 1946.

(73) Dunn, D. W. – Ph.D. thesis, Dept. of Math., Mass. Inst. Tech., 1953.

(74) Dunn, D. W., and Lin, C. C. – *On the Stability of the Laminar Boundary Layer in a Compressible Fluid*, J. Aero. Sci., Vol. 22, p 455-477, July, 1955.

(75) Cheng, S. I. – *On the Stability of Laminar Boundary Layer Flow*, QUART. APPL. MATH., VOL. 11, P 346-350, OCTOBER, 1953.

(76) Lees, L. – *The Stability of the Laminar Boundary Layer in a Compressible Fluid*, NACA REP. NO. 876, 1947; FORMERLY ISSUED AS TECH. NOTE 1360.

(77) Morduchow, M., Grape, R. G., and Shaw, R. P. – *Stability of Laminar Boundary Layer Near a Stagnation Point Over an Impermeable Wall and a Wall Cooled by Normal Fluid Injection*, NACA TECH. NOTE 4037, AUGUST, 1957.

(78) Dunn, D. W. – *On the Theory of the Stability of the Boundary Layer in a Compressible Fluid*, NATIONAL RESEARCH COUNCIL OF CANADA, NAE REPORT, MAY, 1958 (UNPUBLISHED).

(79) Mack, L. M. – *Numerical Calculation of the Stability of the Compressible, Laminar Boundary Layer*, CAL. INST. TECH., JET PROPULSION LAB., REP. NO. 20-122, MAY, 1960.

(80) Laufer, J., and Vrebalovich, T. – *Stability and Transition of a Supersonic Laminar Boundary Layer on an Insulated Flat Plate*, J. FLUID MECH., VOL. 9, P 257-299, OCTOBER, 1960.

(81) Demetriades, A. – *An Experiment on the Stability of Hypersonic Laminar Boundary Layers*, J. FLUID MECH., VOL. 7, P 385-396, MARCH, 1960. SEE ALSO J. AERO/SPACE SCI., VOL. 25, P 599-600, SEPTEMBER, 1958.

(82) Lees, L., and Reshotko, E. – *Stability of the Compressible Laminar Boundary Layer*, PAPER PRESENTED AT BOUNDARY LAYER RESEARCH MEETING, AGARD, NATO, LONDON, ENGLAND, APRIL, 1960.

(83) Pai, S. I. – *Fluid Dynamics of Jets*, P 193-208, VAN NOSTRAND, NEW YORK, 1954.

(84) Lin, C. C. – *On the Stability of the Laminar Mixing Region Between Two Parallel Streams in a Gas*, NACA TECH. NOTE 2887, JANUARY, 1953.

(85) Miles, J. W. – *On the Disturbed Motion of a Plane Vortex Sheet*, J. FLUID MECH., VOL. 4, P 538-552, SEPTEMBER, 1958.

(86) McIntosh, R. F. – *Investigation of Heat-Transfer Effects on Boundary Layer Stability in Water*, UNPUBLISHED. MASS. INST. OF TECH., DEPT. OF MATH., 1954. (RESULTS QUOTED IN REFERENCE 71.)

(87) Lachmann, G. V. – *Fundamental Design Problems of Aircraft with Boundary Layer Control for Maintaining Laminar Flow*, ADVANCES IN AERONAUTICAL SCIENCES, PROCEEDINGS OF THE 1ST INT. CONG. AERO. SCI., MADRID, SEPTEMBER, 1958, VOL. 2, P 587-619, PERGAMON PRESS, 1959.

(88) Schmued, E. – *Progress in Low Drag Boundary Layer Control*, AERO. ENG. REV., VOL. 16, P 34-36, MARCH, 1957.

(89) Lachmann, G. V. (Editor) – *Boundary Layer and Flow Control – Principles and Applications*, 2 VOLS., PERGAMON PRESS, LONDON, 1960.

(90) Czarnecki, K. R., and Sevier, J. R. – *Investigation of Effects of Roughness, Surface Cooling, and Shock Impingement on Boundary Layer Transition on a Two-dimensional Wing*, NASA TECH. NOTE D-417, JUNE, 1960.

(91) Cooper, M., Mayo, E. E., and Julius, J. D. – *The Influence of Low Wall Temperature on Boundary Layer Transition and Local Heat Transfer on 2 Inch Diameter Hemispheres at a Mach Number of 4.95 and a Reynolds Number per Foot of 73.2 $\times$ 10⁶*, NASA TECH. NOTE D-391, JULY, 1960.

(92) Stetson, K. F. – *Boundary Layer Transition on Blunt Bodies with Highly Cooled Boundary Layers*, J. AERO/SPACE SCI., VOL. 27, P 81-91, FEBRUARY, 1960.

(93) Chapman, D. R., Kuehn, D. M., and Larson, H. K. – *Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on the Effect of Transition*, NACA REP. NO. 1356, 1958. (REPLACES TECH. NOTE 3869, MARCH, 1957.)

(94) Siegel, R., and Shapiro, A. H. – *The Effect of Heating on Boundary Layer Transition for Liquid Flow in a Tube*, AMER. SOC. MECH. ENG., PAPER NO. 53-A-178, NOVEMBER-DECEMBER, 1953.

(95) Kramer, M. O. – *Boundary Layer Stabilization by Distributed Damping*, J. AMER. SOC. NAVAL ENGINEERS, VOL. 72, P 25-33, FEBRUARY, 1960. SEE ALSO, J. AERO/SPACE SCI., VOL. 27, P 69, JANUARY, 1960.

(96) Benjamin, T. B. – *Effects of a Flexible Boundary on Hydrodynamic Stability*, J. FLUID MECH., VOL. 9, P 513-532, DECEMBER, 1960.

(97) Haupt, O. – *Uber die Entwicklung Einer Willkürlichen Funktion Nach den Eigenfunktionen des Turbulenzproblems*, SITZUNGSBERICHTE, KÖNIG. BAY. AKAD. DER WISS. ZU MÜNCHEN, MATH.-PHYS. KLASSE, P 289-301, 1912-1913.

(98) Grohne, D. – *Uber das Spektrum bei Eigenschwingungen Ebener Laminarströmungen*, Z. ANGEW. MATH. MECH., VOL. 34, P 344-357, 1954. ENGLISH TRANSLATION, NACA TECH. MEMO. 1417, 1957.

(99) Case, K. M. – *Stability of Inviscid Plane Couette Flow*, PHYSICS OF FLUIDS, VOL. 3, P 143-148, MARCH-APRIL, 1960.

(100) Clauser, F. H. (Editor) – *Plasma Dynamics*, SYMPOSIUM, AIR FORCE OFFICE OF SCIENTIFIC RESEARCH – NATIONAL ACADEMY OF SCIENCES, JUNE, 1958, P 317-319, ADDISON-WESLEY PUBLISHING CO. INC., 1960.

(101) Gregory, N., Stuart, J. T., and Walker, W. S. – *On the Stability of Three-Dimensional Boundary Layers with Application to the Flow Due to a Rotating Disk*, PHIL. TRANS. ROY. SOC. LONDON, SERIES A, VOL. 248, P 155-199, 1955.

(102) Brown, W. B. – *Numerical Calculation of the Stability of Cross-Flow Profiles in Laminar Boundary Layers on a Rotating Disk and on a Swept-Back Wing and an Exact Calculation of the Stability of the Blasius Velocity Profile*, NORTHROP AIRCRAFT INC., REP. NO. NAI-59-5 (BLC-117), JANUARY, 1959.

(103) Spence, D. A., and Randall, D. G. – *The Influence of Surface Waves on the Stability of a Laminar Boundary Layer with Uniform Suction*, ROYAL AIRCRAFT ESTABLISHMENT, TECH. NOTE AERO. 2241, MARCH, 1953. (AERONAUTICAL RESEARCH COUNCIL, CURRENT PAPER NO. 161, 1954.)

(104) Lin, C. C., and Rabenstein, A. L. – *On the Asymptotic Solutions of a Class of Ordinary Differential Equations of the Fourth Order. Part I, Existence of Regular Formal Solutions.* TRANS. AMERICAN MATH. SOC., VOL. 94, P 24-57, JANUARY, 1960.

(105) Thomas, L. H. – *The Stability of Plane Poiseuille Flow*, PHYS. REVIEW (2), VOL. 91, P 780-783, 1953.

(106) Benney, D. J., and Lin, C. C. – *On the Secondary Motion Induced by Oscillations in a Shear Flow*, PHYSICS OF FLUIDS, VOL. 3, P 656-657, JULY-AUGUST, 1960.

(107) Klebanoff, P. S., and Tidstrom, K. D. – *Evolution of Amplified Waves Leading to Transition in a Boundary Layer with Zero Pressure Gradient*, NACA TECH. NOTE D-195, SEPTEMBER, 1959.

MAN-POWERED FLIGHT – A MYTH OR REALITY?†

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SUMMARY

In writing this article, the author's intention is to present to the reader the highlights of the problem of designing and building a man-powered aircraft as seen by an aircraft designer today.

An attempt has been made to explain, first, why all the previous attempts to build a successful man-powered aircraft failed and, consequently, to show the reader how difficult it is to make an aircraft a complete success. In the first part of the article the beneficial effect of increasing the number of slave-passengers upon the efficiency of such aircraft is discussed.

Further, an attempt is made to propose and justify the minimum strength requirement for a man-powered aircraft in view of the mission it should perform. Based on these strength requirements, the sizes and weights of a large family of two seater aircraft ranging from 150 to 350 sq ft wing area and wing aspect ratios of 5 to 30 are estimated and the aerodynamic characteristics of an optimum design with respect to a minimum power requirement is found.

As a result of this optimization, a critical analysis of the quantities involved in view of the expenditure and distribution of the available amount of power is made, and some obvious avenues of possible improvement are discussed.

It is suggested that, in order to design a fully successful aircraft, more research and test work must be done at properly equipped institutions, in the field of low drag aerofoils at low Reynolds numbers, as well as on highly efficient slow running propellers and transmissions. Only then, a contemporary aircraft designer equipped with a knowledge of modern materials and design methods could succeed in this challenging endeavour.

APPRECIATION OF THE PROBLEM

For anyone who becomes interested in man-powered flight and who intends to learn more about it, there is a good deal of information available. The pertinent material consists mostly of articles covering various aspects of the problem written in many languages, a number of reports describing the results of research and testing mostly on the measurement of the available human power, as well as reports on design, construction and attempts to fly full scale aircraft. In particular, two cases of full scale single seater designs are of great interest, namely the German design by Haessler-Villinger and the Italian Bossi-Bonomi, built and flown in 1935 and 1937, respectively.

In spite of the fact that the attempts made so far have not been successful, there still exists plenty of interest in man-powered flight. The old unfulfilled dream to fly like a bird, which acquired perhaps its nearest approximation in gliding, is still not fully satisfied. An aircraft powered by man's own power still remains very alluring and challenging, in spite of the highly motorized life which we all lead today (and do not always like).

Why have all attempts made so far to build and fly a man-powered aircraft been unsuccessful? In order to give the reader a somewhat simplified answer to this complex question, it is instructive to make a simple comparison between a man-powered aircraft and a bicycle. Such comparison is valid only as far as it concerns the human motor propelling both types of vehicles and the demand of power which each of these vehicles requires from the driver.

We have learned from experience what performance to expect from a bicyclist and his bicycle. On the other hand, knowing the aerodynamic and mechanical performance of the Haessler-Villinger or Bossi's aircraft, we can carry our comparison quite far, and can obtain a fairly clear explanation of their lack of success.

Let us limit our comparison to one argument only, that is the power requirement, and analyze the amount of effort expended by a cyclist in driving his bicycle uphill. First of all, we know how much the cyclist and his bicycle weigh and at what speed he is able to drive it. From straight observation and from our own experience of driving bicycles we have developed a fairly good idea about the sort of performance to be expected.

What sort of a contraption, in terms of a bicycle, does a man-powered aircraft represent? For the sake of comparison, let us consider here a man-powered aircraft having similar characteristics to the Haessler-Villinger machine. We know its weight and we know how fast it has to be moved through the air to stay airborne. Knowing the aerodynamic characteristics of the aircraft and the efficiency of its propulsion system we can easily calculate the amount of "uphill" the driver of such an aircraft must constantly climb in order to fly horizontally.

The figures used for our calculations are:

Total weight of aircraft and pilot	245 lb
C_L/C_D = lift-drag ratio at optimum power incidence	22.5
Wing area	104 sq ft
Wing aspect ratio	18:1
Wing loading	2.35 lb/sq ft
Flying speed at minimum power	41.0 ft/sec
Total mechanical efficiency of the propulsion system	78%

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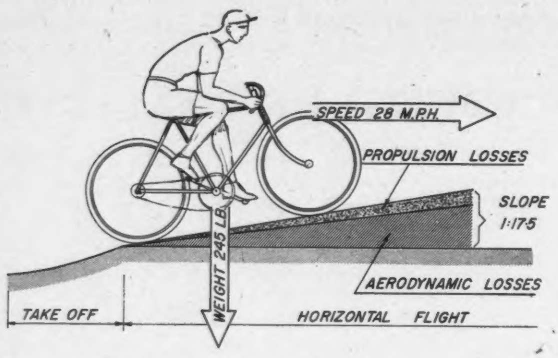

Figure 1
Comparison between a cyclist and a pilot driving a man-powered aircraft

In Figure 1 the results of our comparison are presented. From looking at the diagram it becomes quite apparent that in order to fly horizontally a man-powered aircraft of the Haessler-Villinger type would be equivalent to driving a bicycle which would weigh about 100 lb, continuously uphill at a slope of no less than 1:17, and at a speed not less than 28 mph.

A very simple check proves that to fly an aircraft of similar characteristics requires about 1 hp. The performance obtained by Haessler was such as to prolong an almost horizontal flight by several hundred feet after the aircraft was catapulted into the air by a shock cord or a motor car.

Full recognition of Haessler's effort should be given here for the excellence of the design of his aircraft. The general layout and the small weight achieved at the ultimate load factor of 6 were really remarkable. But our poor air-cyclist was asked to do work far exceeding his physical ability. His physical strength was not enough to cope with such an aircraft. Apparently everything, the weight, the mechanical efficiency of the propulsion system and the speed, was far beyond that which a human is accustomed to when driving a bicycle.

What can our cyclist really do? A complete knowledge of the human as a source of mechanical power has to be considered as a most important key to the problem of a successful man-powered aircraft, and the whole design of it should be tailored to meet the specifications of a human motor.

Unfortunately, there is not enough statistical material available to give us a complete answer. A most important and reliable source of information comes from Ursinus[1] and his efforts to establish the optimum conditions for delivery of human power. The limitations of his work are that the range of his tests extended only up to 170 seconds, and that all his results were obtained from people of average physical ability and do not include professional sportsmen. For instance, it is a well known fact that Bossi's pilot, Emilio Casco, who was both a pilot and a professional cyclist, gave a far superior performance to those published by Ursinus, when tested by Ursinus in his Man Power Flight Institute in Darmstadt. For a designer it would be very important and useful to have available reliable data on record human power output as well as on average. We should not forget that the future achievements of man-powered aircraft will be a combined achievement of the physical performance of the pilot or pilots and of his flying machine.

Figure 2 shows performance curves based on results of the classic work of Ursinus. On the graph the author has purposely discriminated between the work done by legs only and by both hands and legs of the pilot. In the case of a single seater all the power to drive the aircraft must be obtained from his legs, as his hands would be fully occupied controlling the aircraft. In the case of a two seater (or higher number of seats), all the other passengers except the pilot can work with their hands and legs, thus increasing very considerably the total power output, especially within the short-time range, which can be extremely valuable during takeoff. All the power curves shown for the larger number of passengers in Figure 2 are based on this assumption.

The next question which comes logically is the question of how refined an aircraft must be in order to suit the available man-power. This question cannot be answered easily, as there are too many variables affecting the subject. In order to simplify the problem only three variables have been considered: the aircraft speed, the weight and C_L/C_D ratio at minimum power incidence. Of these three variables, two can be further eliminated by assuming some typical constant values for speed and weight which have been found to be most realistic and probable, and which will be justified later. The horizontal flight speed assumed is 40 ft/sec, which corresponds to a wing loading of about 2 lb/sq ft and $C_L = 1$. For the weights of the aircraft, some average weight figures obtained from a detailed weight analysis have been used, except in the case of a one seater, where a weight similar to the Haessler-Villinger aircraft has been considered.

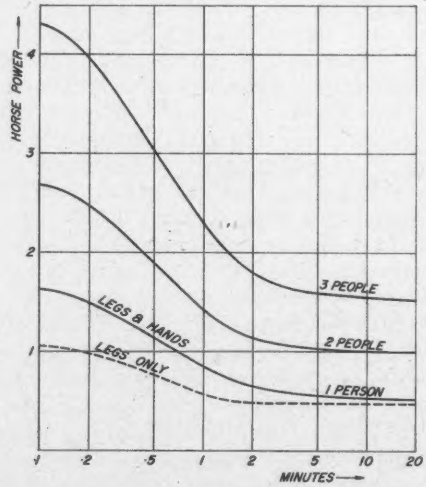

Figure 2
Man-power output vs time

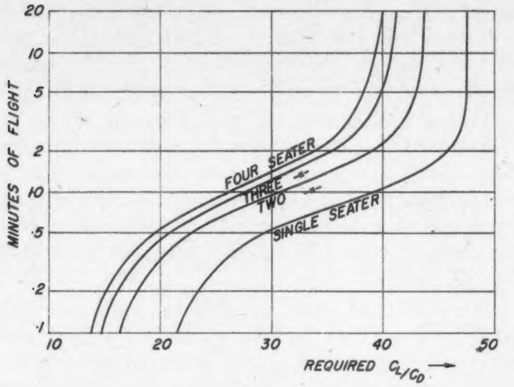

Figure 3
Flight duration vs C_L/C_D required for a horizontal flight

In particular, the following average empty weights have been used:

One seater	100 lb
Two seater	180 lb
Three seater	246 lb
Four seater	301 lb

The weight of one pilot has been assumed to be 150 lb, and a constant propulsive efficiency of the propeller system plus mechanical drive has been assumed to be 80%. This figure is rather high by present standards, but the author believes that with a good design of mechanical drive and large diameter propellers such an efficiency would be realistic.

Figure 3 represents the results of combining the power curves shown in Figure 2 with the above weights and constant flight speed of 40 ft/sec.

Of course, the results presented here should be considered as approximate since they would apply only for aircraft having wings within the aspect ratio range of 20 to 25 and about 2 lb/sq ft wing loading, for which the assumed weights are realistic.

The main conclusion from Figure 3 is that the aircraft must fly at a very high C_L/C_D ratio in order to stay airborne for a reasonable length of time. Such high C_L/C_D values can only be obtained for aircraft which are very clean aerodynamically, and which would be flown close to the ground to minimize drag due to lift.

Another assertion is evident. The one seater has a very small chance of becoming a successful aircraft. There are two reasons for this. One is that the power available from the pilot cannot be fully utilized for propulsion since his hands have to control the aircraft. The second reason is that the ratio of available power to the empty weight of the aircraft would be always less favourable than for a multi-seater. The gain in flying time resulting from the larger number of seats is very spectacular. For instance, at a C_L/C_D ratio of 40, which probably is quite feasible to obtain when flying within the air cushion, the four seater would be able to fly 20 times longer than the one seater.

It seems that there is no optimum number of seats, as it is obvious that the flying time will always continue increasing with number of seats for the same C_L/C_D value. For larger numbers of seats the aircraft must be larger and this in turn will cause an improvement in weight and in aerodynamic performance due to the increased Reynolds number (which might mean quite appreciable reduction in profile and parasite drag). Figure 3 is illuminating in that it shows very clearly how refined an aircraft must be in order to give the expected performance. It may be that the results shown here are a little pessimistic, as they were based on Ursinus' curves for the "average" human performance. How much better the results would be for the "record" type of sportsman still has to be investigated. Comparing the results of the full-scale experiments of Bossi and Haessler-Villinger, one immediately perceives how much the human factor matters. Indeed, to design and build an aircraft which would be clean enough and light enough to be flown for a reasonably long time by muscle force alone would be a formidable engineering endeavour.

Therefore, the next logical question which requires an honest answer is "How good an aircraft could be offered today to the enthusiast who would be willing to toil heavily for the pleasure of flying purely by his own power?"

STRUCTURE AND WEIGHT

Strength requirements

As mentioned and proven earlier, the aerocycle has to be a very clean aircraft. It must also be a very light aircraft. However, the author disagrees with the statement frequently made by other authors that it has to have a very low limit load factor. After all, it has to be a practical aircraft capable of being flown safely. It may easily be that such an aircraft will occasionally fly quite high.

Each aircraft is designed and built to serve a specific purpose or to fulfill a certain mission or number of missions. Therefore, before starting a discussion on the required strength, we should ask "What sort of mission would a man-powered aircraft be expected to fly?"

It is certain that relying on man-power alone would excessively limit the potential uses of such an aircraft. It would also be very unappealing and tiresome for the flying crew to know that they would be able to fly only as long as their physical endurance permitted. It is only natural that they would immediately start looking for some other source of power in order to fly longer and use the human power only as long as it would be necessary to find uplift. It would be very easy to catch a thermal when flying an aircraft as clean and light as the one being considered.

It is worthwhile perhaps to mention some flight cases. For example, a sailplane made a very spectacular soaring flight above a paved highway during a hot and sunny day. The difference in temperatures between the pavement and surrounding fields was sufficient to create an uplift strong enough to sustain a sailplane. The author remembers, from his own gliding experiences, some instances on calm, sunny days when very large areas of rising air were observed to be strong enough to sustain a glider at fairly low altitudes.

All the hitherto proposed strength requirements do not take into account the possibility of flying higher when using outside sources of energy. Usually they only considered flights powered by the work of human muscles, and a typical claim has been that man-powered flight has to be only a "calm weather sport".

Why restrict from the very beginning the potential possibilities of a promising and attractive sport? On the other hand, the experienced aircraft designer knows that in order to increase the load capacity of an aircraft by an appreciable percentage very little weight has to be added to the original structure. This would be especially true in the case of a low density structure, where the bulk of the airframe weight would come, for the most part, from the necessity of satisfying the local structural stability as well as keeping the geometric exterior shape true enough to assure the required high aerodynamic cleanliness.

High aerodynamic cleanliness will also require high aspect ratio wings, and a too low load factor may prove troublesome because of inadequate stiffness. For example, the author calculated the deflection of a cantilever tapered wing for a two seater having a wing area of 170 sq ft, aspect ratio of 22.5 and a taper ratio of 5:1, which was designed to meet an ultimate load factor of 2.5, as proposed by some authors. At an aerofoil thickness of 12% and normal flight conditions, at limit load factor $n = 1$, the wing-tip deflection would be of the order of 70 in. Would such a wing be practical?

The proposed average man-powered aircraft mission will comprise flights which would be started by man-power alone for the purpose of finding, as soon as possible, enough energy in the atmosphere to prolong the flight at will. Usually, mild thermals or slope currents will be sufficient to sustain the aircraft's flight. The possibility of abandoning one source of power and looking for another when using man-power alone would add zest and excitement to this new sport. Because of a very low wing loading the flights would usually be undertaken in fairly good weather, free of strong winds and/or gusts.

Wherever the air is ambient there are bound to be gusts. However, it is the magnitude of the gusts which would be of specific interest and importance, and it will be necessary to establish by statistics the maximum possible gust magnitude and the required minimum strength of such an aircraft to make it safe for the specified mission or operating regime.

Further, it is necessary to consider a flight condition whereby a man-powered aircraft has to be brought down for landing from a reasonable altitude. This manoeuvre will require an increased speed, the magnitude of which will be a very important factor in the design of the aircraft. It may easily be that efficient spoilers will prove necessary and become standard safety equipment for this type of aircraft.

Prior to official airworthiness regulations being issued by the proper authorities, it would be useful to discuss some minimum strength requirements which would fit the flight mission just described.

In order to propose a V/n diagram for the man-powered aircraft, two basic quantities have to be settled. The first is the magnitude of the "n" factor and the second is the magnitude of the never exceeded design speed, which is usually equal to the maximum permissible diving speed (at $C_L = 0$).

When considering the possibility of flying such an aircraft at reasonable altitudes, say from 0 to about 1000 ft, in fairly good weather, the author does not see much justification for reducing by a large margin the standard strength requirements specified by the British Civil Airworthiness Authority for the "normal" class of gliders.

For the above class the specified minimum design speed is 3 times the stalling speed. Assuming, for instance, in the case of a man-powered aircraft a reduced design speed of 2½ times the stalling speed would be a very "mild" requirement indeed for an aircraft which has to be a very clean aircraft aerodynamically and will have a natural tendency to gain speed very rapidly when pitched. As the calculations show, the speed equal to 2½ times the stalling speed would be in most cases only about 5% less than twice the speed corresponding to the best gliding angle.

Reducing the maximum permissible diving speed from 3 to 2½ times the stalling speed would also reduce the design wing torsional loads by about 28% which would mean a considerable relaxation of requirements as compared with gliders and would greatly assist in an appreciable weight reduction.

Based upon his own design experience, the author believes that a minimum design speed approximately twice the speed at the best gliding angle would be the minimum practical requirement and still produce a safe and sufficiently rigid aircraft to fit the proposed operating regime.

The positive limit manoeuvring load factor specified for the normal class of gliders is $n = 4$. How much it could be reduced in our case would depend upon the kind of weather in which it would be allowed to fly and the maximum speed at which it would be fully manoeuvrable. For instance, requiring full manoeuvrability to be fully retained at, say, 1.75 to 1.85 times the stalling speed, would result in a limit load factor approximately between 3 and 3.4. Choosing a reduced limit load factor equal to 3.33 would yield an ultimate load factor equal to 5, corresponding to a flight speed of 1.825 times the stalling speed, which would only be about 25% greater than the speed at the best gliding angle. Again, this figure seems to be a reasonable requirement taking into account the flying speed which might be used quite often.

The negative load factor of 1.25 should be suitable as every aircraft should be able to fly in an inverted position free of danger of disintegration. A very "mild" manoeuvring load factor of 0.25 on top of a normal inverted flight load would again be well justified. This requirement, as we know, is usually only a formal requirement as any wing spar designed to withstand a positive limit load factor of 3.33 would be strong enough at a negative load factor of such magnitude without making it necessary to put any additional weight into the load carrying structure.

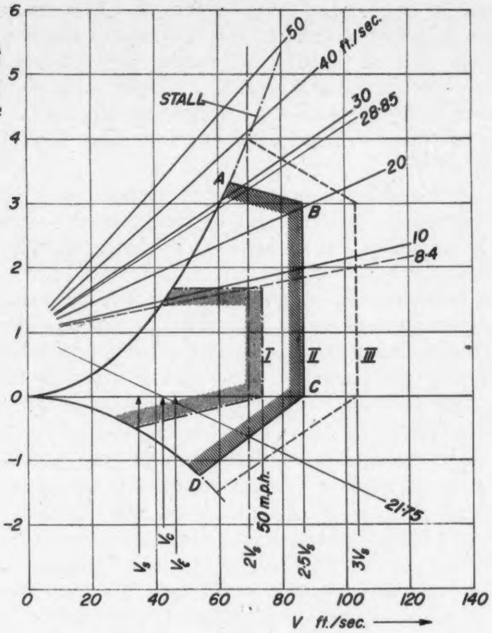

Figure 4
***V/n* diagrams for a man-powered aircraft**

Figure 4 (diagram II) shows a sample *V/n* diagram calculated for a man-powered two seater which the author considers typical. The main characteristics of such an aircraft are:

Wing area	250 sq ft
All-up weight	547 lb
Wing aspect ratio	25
$C_{L\,max}$	1.53
$C_{L\,cruise}$	1.04
$C_{L\,\text{best gliding angle}}$	0.87
$dC_L/d\alpha$	5.715

The proposed flying mission will inevitably expose the aircraft to the effects of gust. From Figure 4 (diagram II) it is seen that the highest gust velocities which could be met at the proposed load factors and a speed twice the stalling speed are only 28.5 ft/sec positive and 21.75 ft/sec negative. This will place quite a limitation on the kind of weather in which our aircraft would be allowed to fly.

For comparison, two more *V/n* diagrams have been shown in Figure 4. Diagram I represents a *V/n* diagram which has been frequently proposed by other authors and designers. As is apparent from the graph, such low requirements would only be valid for a positive gust not greater than 8.4 ft/sec, which could hardly be considered practical.

Diagram III in the same figure represents a standard *V/n* diagram as specified by the British Airworthiness Requirements for the "normal" type of glider. One can easily perceive that the requested standard gust velocity of 50 ft/sec is irrelevant at speeds below twice the stalling speed of the aircraft.

Structural problems

The planform of the wing has a significant effect upon the aerodynamic and structural characteristics of an aircraft. Many different aspects should be considered such as cost of production, structural wing stiffness and weight, induced and profile drag and so forth. Usually the problem of the most suitable planform shape is a complete study in itself in order to arrive at the best compromise for a specific purpose. For instance, from the standpoint of minimum over-all wing drag, the elliptical planform may not prove the best at the Reynolds number range at which a man-powered aircraft will operate. At very high aspect ratios, the very small wing tip chords may cause an appreciable increase in profile drag and loss of lift. These effects may become more detrimental than the small increase in induced drag caused by deviation from an ideal planform. In the following investigation the assumed planform of the wing is substantially rectangular, having a constant chord and a long straight trailing edge. The aerodynamic values and merits of such a wing are almost as good as those for an elliptical wing[2] and may prove beneficial in the utilization of ground effect.

Simple investigations prove that a simply braced wing, using profiled steel strip, of thin cross-section, might be the most economical answer to the problem of cutting down wing weight and improving stiffness. Thus, a simply braced wing, supported at the middle of the half-span by a single profiled steel strip, is considered in the following evaluation of weight.

In the case of very low density structures, usually the greatest difficulty arises from the problem of proper distribution of the structural material in such a manner that the local stability of load carrying structural elements would be satisfied. Therefore, low density structural materials should be used wherever possible as they would give the largest possible cross-section at the same weight. In the case of a high aspect ratio wing, an adequate bending strength will always be a major factor affecting weight. Hence, the bending structural material should be located farthest from the neutral bending axis. A most efficient structure would be a classical D-nose-spar having all bending material located at the section periphery. The variation of bending moment along the wing span could easily be taken care of by changing locally the thickness of the bending material or varying its strength. Such a D-nose-spar would be very light indeed and would offer the best possible strength and stiffness characteristics in both the horizontal and vertical planes. The leading edge would also assume a very smooth and stable shape, which may be important in achieving a low profile drag.

In order to create satisfactory torsional strength and stiffness in such a spar, the author proposes to cover it with layers of fibreglass fabric arranged at 45° to the longitudinal axis of the spar. Thus, an efficient structure would be created which could be scaled exactly to the local bending moment and torque requirements. Where a higher rigidity in torsion is required, additional layers of glass fabric could be cemented on top of each other, building up the required thickness. Figure 5 shows a typical section through such a spar structure. As calculations prove, such a structure would be much more efficient than

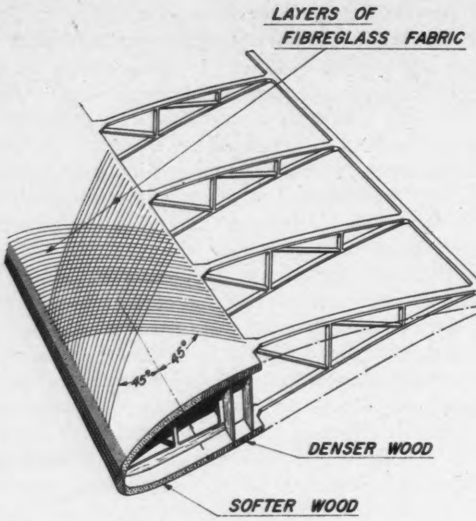

Figure 5
Proposed wing structure

the classical construction of a single spar taking all the bending, and a plywood-covered D-nose. Weight-wise, for the same stiffness, a plywood-covered D-nose would be about 30% to 60% heavier than the D-spar covered with fibreglass. The main deficiency of the plywood-covered D-nose is its lack of efficiency at higher stresses due to buckling. A plywood-covered D-nose having the outer fibres arranged at 45° to the spar axis, in order to give a maximum torsional rigidity, is inefficient in bending due to its low modulus of elasticity in this direction.

A D-spar of the type shown in Figure 5 should give the designer a flexible and convenient design scheme, which could be tailored to match any strength and stiffness requirement.

Structural weight

Having the load factors and the type of wing structure reasonably defined, the estimation of wing weight for various aspect ratios and wing areas becomes simply a matter of straightforward calculation. Dividing the whole wing structure into two groups, namely the load carrying structure and the structure needed to hold the geometrical shape, a fairly detailed estimate has been made for a family of two seater wings having a similar 12% aerofoil but varying aspect ratio from 5 to 30 and wing area from 150 to 350 sq ft. The results obtained are shown in Figure 6, and they check well with past man-powered aircraft experience.

By carrying out a similar procedure for the tailplane and fuselage, taking into account the variation of tailplane size with wing aspect ratio and optimum length of the tailplane cone, a fairly complete appreciation of weight variation has been obtained. Adding to it some constant quantities such as the weight of pilot's cockpit, undercarriage and propulsion system, the author arrived at the final results as given in Figure 7. The author does not claim that the empty weights of the complete aircraft, as indicated, are exact or could not be improved upon. In our case, the greatest importance of the weight estimates given here should be related to a proper appreciation of the rate of change of weight with variation of wing aspect ratio and area. The weight variation parameter is most vital in the following attempt to find the optimum size and basic characteristics data of the aircraft.

In the case of a man-powered aircraft, every piece of structural material and hardware used has to be fully justified and as light as possible. Equally important is the initial layout and the skill which the designer exercises in preparing the detailed design of components and parts. The initial layout and relative arrangement of components have to be studied carefully in order to arrive at a solution which would be the best compromise from the standpoint of structure and weight.

It is impossible in this paper to give a detailed description of how to build a successful man-powered aircraft. It can be said, however, that all parts and components should be arranged in such a manner as to produce an aircraft of the greatest possible aerodynamic and mechanical excellence. The least possible drag of non-lifting components, interference drag, effect of propeller or propellers upon the aerodynamic drag of wing and tailplane and the best utilization of ground effect are only a few of the outstanding problems involved.

AERODYNAMIC ASPECTS

After establishing the relation between the weight of the aircraft and a wide range of wing areas and aspect ratios, an attempt has been made to find the

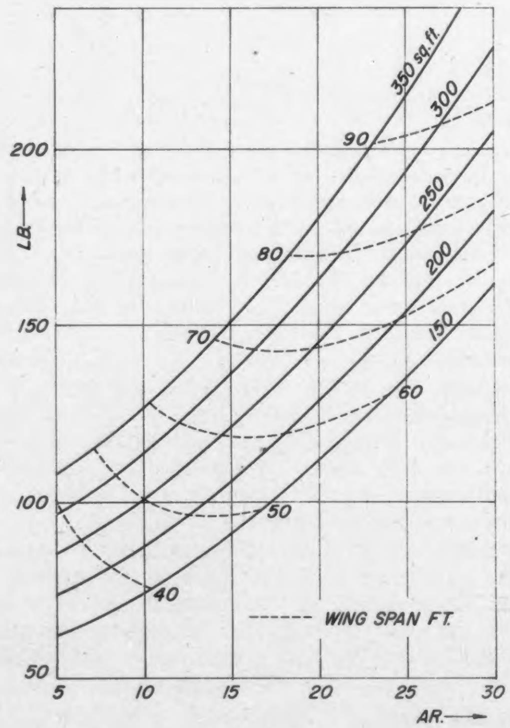

Figure 6
Weight of the wing vs wing area and aspect ratio

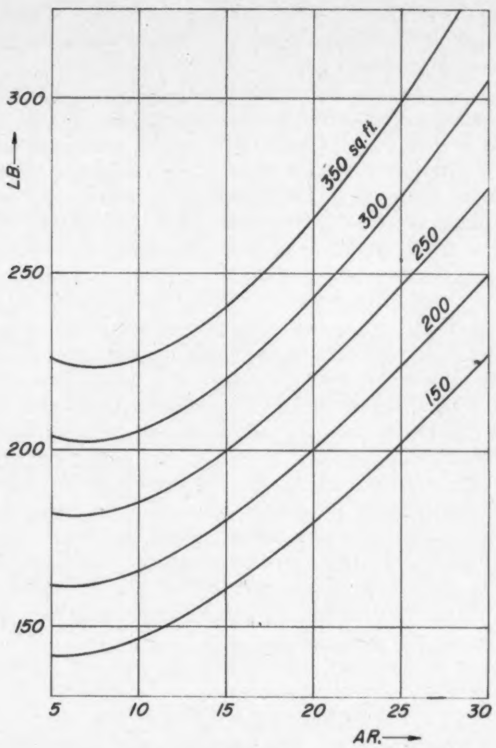

Figure 7
Empty weight of a two seater man-powered aircraft vs wing area and aspect ratio

optimum size and shape of an aircraft which would require minimum power for horizontal flight.

In order to do this, the first step had to be to find a good aerofoil for the aircraft wing. This task was not easy, as there is really very little material available which would supply the aerodynamic properties for a choice of aerofoils for the Reynolds number at which the man-powered aircraft would operate – which most probably would range between 0.6 and 1.4×10^6 depending upon the wing chord and speed.

The information which the author found most valuable for this further work was Reference 3, which deals mostly with the NACA 4- and 5-digit aerofoil series tested in a variable density wind tunnel over a fairly large range of Reynolds numbers. But even there, the turbulence correction factors relating the test Reynolds numbers with the effective ones measured in flight, as well as correction increments relating the test and the effective profile drag coefficients, could not be considered fully reliable for our case; as the recommended values for these factors were averaged for a range of Reynolds numbers much higher than the ones which would be encountered in man-powered flight.

The test results published in another NACA report[4] which deals with more modern 6-digit NACA aerofoil sections were not used for further optimization, as the author does not believe that the so-called "smooth surface conditions" essential for such aerofoils could be practically obtained on a full-scale wing for such a lightly built aircraft. One should not forget here that the smooth surface condition does not mean only the degree of surface finish but pertains also to the highly accurate reproduction of the proper geometrical shape of the aerofoil.

The best method for comparing aerofoils from the standpoint of minimum power required is to plot the coefficients $C_D/C_L^{3/2}$ as a function of incidence or even more conveniently as a function of lift coefficient. Such a plot is shown in Figure 8 which was prepared for the four 12% thick NACA aerofoils belonging to the 4-digit series and having a camber ranging from 0 to 6% at 40% of the chord. As can be seen from the graph, the best aerofoil found on this basis for about 1.2×10^6 Reynolds number is NACA-6412 which was chosen as a sample aerofoil for all the following calculations.

From the cross-plotting of the polar diagrams obtained from tests at various Reynolds number for the 6412 aerofoil, the relation between the profile drag, lift coefficient and the Reynolds number was found and is presented in Figure 9. However, to save work in the following computations, a Reynolds number multiplied by the kinematic viscosity ν was used as the horizontal co-ordinate in order to obtain a dimension of ft²/sec, which would correspond directly to the product of the wing chord and the horizontal speed.

Further, the relation between the weight, wing area and aspect ratio found in the previous section was used for calculating the aircraft's horizontal speed as a function of C_L. Finding the product of the mean wing chord and speed of the aircraft at various C_L

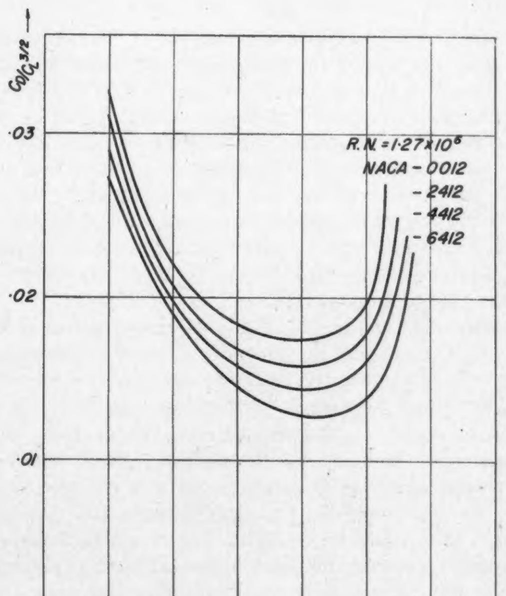

Figure 8
Comparison of four NACA 4-digit aerofoils tested at Reynolds number 1.27×10^6

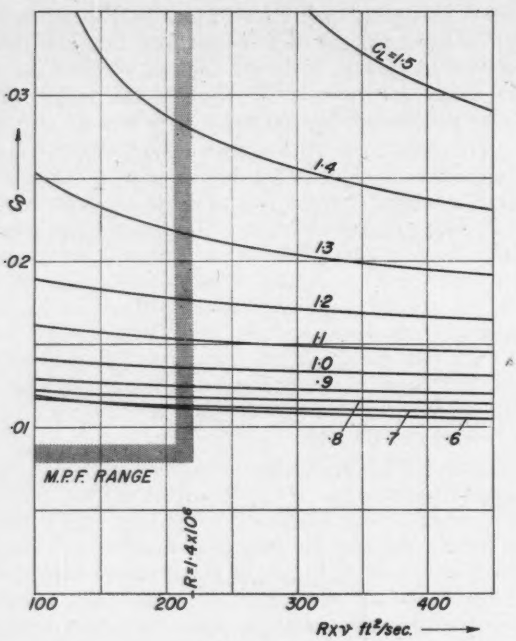

Figure 9
Profile drag of NACA-6412 vs lift coefficient and Reynolds number

values and aircraft configurations, the proper figure for the profile drag could be located from Figure 9.

Thus, a proper correlation between the profile drag and the horizontal speed was established, which enabled the effect of the variation of profile drag with Reynolds number to be included in the calculation of aircraft performance.

A routine procedure was applied in evaluating the parasite drag as well as the induced drag for the fuselage, tailplane and wing bracing. Also the fuselage interference drag, as well as the induced drag of the tailplane, was found as a function of wing incidence, or more conveniently in our case, as a function of C_L.

The minimum value for the total parasite drag reduced to the corresponding wing area was shifted near a lift coefficient of $C_L = 0.9$ which was found to be a good approximate value for the C_L at which the minimum power required for flight usually occurs. In practice this condition could be easily achieved by rigging the wing at the proper incidence with respect to the fuselage. Figure 10 shows the parasite drag coefficients as a function of wing area and C_L.

As pointed out before, bracing of the wing was considered necessary, as the author believes that the very high aspect ratio wings typical for man-powered aircraft would not be practical without bracing, because of prohibitive weight and flexibility. When properly designed, the bracing would add very little to the wing drag, as it would increase the drag area only by about 0.09 sq ft. The proposed bracing would be of flat steel strip having a section of about 0.5×1 inches with sharpened leading and trailing edges. Such bracing should perform very well, as its drag should be not greater than that of a flat plate. The Reynolds number at which such bracing would operate would be of the order of 22,000 which is already in the Reynolds number range where the flow becomes laminar.

As a result of these rather lengthy and tedious calculations a final graph has been prepared (Figure 11) which shows the relation between the minimum net power required to sustain horizontal flight as a function of wing area and aspect ratio. The results thus obtained are very instructive. They show firstly that for a given family of aircraft of similar basic design having the same wing section and area, as well as the same drag area of the non-lifting components, there exists a definite optimum configuration with respect to the wing aspect ratio at which the minimum power requirement occurs. However, the curves are quite flat within the vicinity of the minimum, which means that deviating from the optimum wing aspect ratio both ways does not greatly affect the power required.

A similar observation can be made with respect to the variation in wing area. Increasing the wing area at the higher wing loadings causes, at first, a very significant minimum power reduction. But a further increase of area in the vicinity of 250 to 300 sq ft improves the power required only slightly. For instance, by increasing it still further from 300 to 350 sq ft the minimum power is reduced only by about 1.8%.

No further calculations with increasing wing areas have been attempted, as it becomes quite obvious from the rapidly converging relative distance between curves representing various wing areas that no further gain in power could be obtained.

One may question the applicability of such a detailed calculation with respect to all future development in the field of man-powered aircraft. However, it can be proved that the optimum dimensions found here, such as the wing loading and aspect ratio, do not change much with variation of the parameters used here. This means that a man-powered two seater will

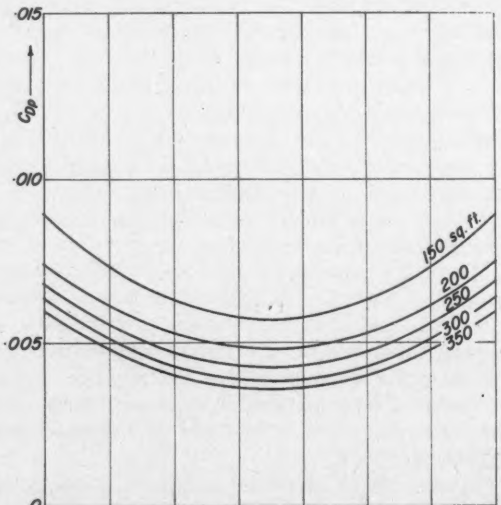

Figure 10
Parasite drag coefficients for a man-powered two seater vs wing area and lift coefficient

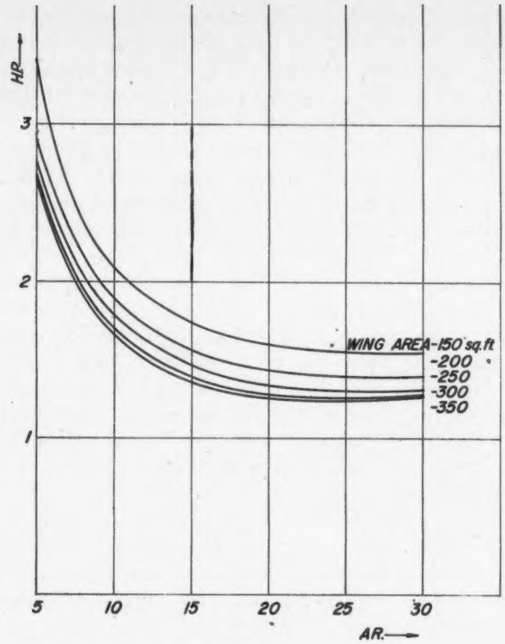

Figure 11
Net hp required to fly a man-powered two seater vs wing area and aspect ratio

be always a comparatively large aircraft having a wing area of the order of 200 to 300 sq ft and an aspect ratio somewhere between 20 and 25. Such an aircraft will have excellent flying qualities at very low altitudes, due to the large wing span which would be able to utilize the ground effect to a very pronounced degree.

The author expects that some very interesting conclusions may be obtained from a closer study of the optimum conditions governing the full utilization of ground effect. When the flying altitude becomes relatively small with respect to the wing chord, the so-called "ram-wing" effect[5] may further reduce the induced drag. More theoretical and experimental work is required in order to evaluate quantitatively such an effect.

The attempted optimization presented here has been made for a rather arbitrarily chosen aerofoil section. How much could the performance of such an aircraft be improved by replacing a conventional aerofoil with a more sophisticated one? In particular, the low drag laminar flow wing sections should be thoroughly discussed and examined.

Unfortunately, there is very little material available which would give us enough information on the laminar flow wing sections in the Reynolds number range between 0.5 and 1.4×10^6. From the data given in the technical reports of the NACA[4] only a very selected number of aerofoils had been tested within the Reynolds number range from 0.7 to 9×10^6, and this was about the only reference the author could find for further calculation purposes.

As is evident from this report, a low Reynolds number has a beneficial effect upon the width of the low drag portion of the polar of a laminar flow section, but the magnitude of the minimum drag coefficient increases very rapidly with decrease of Reynolds number. For the smooth NACA 6-digit aerofoil series the amount by which the drag coefficient increases with lowering Reynolds number appears to become larger as the thickness ratio of the aerofoil increases. This trend indicates that the full advantage in drag reduction could be obtained only by using thinner laminar sections. In our specific case the optimum thickness would be somewhere around 12%. Much thinner sections would lose their practicability due to the rapidly increasing wing weight and loss of stiffness.

It can be seen further from the test data available for the low range of Reynolds numbers that in the case of the NACA 6-digit section series the drag outside the relatively flat portion of the polar increases very rapidly, especially for the aerofoils in the smooth surface condition. This undesirable property narrows very pronouncedly the range of the lift coefficients at which economic flight could be performed.

The drag data for the NACA 4-digit series in smooth surface condition do not show generally as much variation in drag with Reynolds number as do the NACA 6-digit sections. This phenomenon, together with the proven low sensitivity of the NACA 4-digit series to changes in Reynolds number at its lower range, casts doubt on the advantage of using the available laminar aerofoils for a man-powered aircraft. Based on the published test results, it seems that the laminar flow thickness distribution may not be the best solution for a low drag aerofoil in this specific Reynolds number region. For instance, in the case of some NACA 4-digit aerofoil series the position of the peak minimum pressure ahead of the incipient laminar separation point may result in no region of separated flow existing.

As the Reynolds number decreases the extent of laminar flow increases, causing reduction in minimum drag. On the other hand, the extent of laminar flow on the NACA 6-digit sections is limited by the position of laminar separation, which of course does not vary with Reynolds number.

This is illustrated in Figure 12 where the two NACA 6-digit sections having different cambers are compared with a smooth NACA-4412 section. It is noticeable that at the Reynolds number of 1×10^6 the latter is almost as good as the 64,-612 and better than the 64,-412. However, in the case of the 4412 section the effect of the low drag region extends quite pronouncedly into the higher lift coefficient range, which should be considered as a highly desirable feature in flight.

Nothing has been said as yet about the sensitivity of true laminar sections to the accurate reproduction of the proper section geometry. It is very doubtful that in the case of a very low density wing structure a satisfactory accuracy could be maintained. The only way of finding out the magnitude of this effect would be to test in full-scale a portion of a wing simulating closely a real man-powered aircraft structure. In Reference 6 the reader will find a good article written

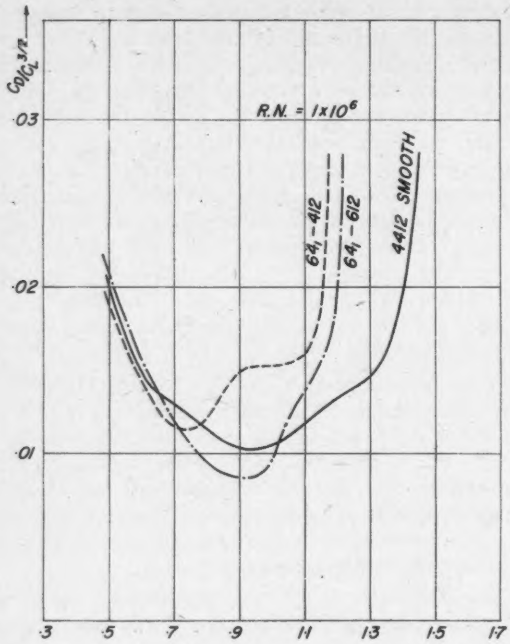

Figure 12
Comparison between three NACA aerofoils of smooth surface condition

by Raspet which discusses the application of low drag aerofoils to sailplanes and which throws more light on this subject, although in the case of sailplanes the effective Reynolds number range is much higher.

In order to obtain a more complete insight into the manner in which the power required is being consumed, a further analysis was made. Consider for instance the formula for power required in horizontal flight:

$$P_{HP} = \frac{WV_S}{550}$$

where W = all-up weight of the aircraft, lb.
V_s = sinking speed, ft/sec.

Expressing the same formula in another form by substituting the expression for V_s we obtain for flight at sea level:

$$P_{HP} = 0.0527\,\frac{W^{3/2}}{A^{1/2}} \times \frac{C_D}{C_L^{3/2}}$$

where C_D is the drag coefficient of the whole aircraft. Breaking down further the total drag into three basic types of drag, namely, induced drag, parasite and wing profile drag, we obtain:

$$C_D = C_{D_i} + C_{D_{pa}} + C_{D_{pr}}$$

Putting this sum in the formula for power required it assumes the final form:

$$P_{HP} = 0.0527\,\frac{W^{3/2}}{A^{1/2}} \times \frac{C_{D_i} + C_{D_{pa}} + C_{D_{pr}}}{C_L^{3/2}}$$

In the case of a specific aircraft of given weight and wing area, the power required for horizontal flight is a function of $C_D/C_L^{3/2}$ only.

If, for instance, the cruising flight occurs at $C_L = 1$ or close to it, which is true in the case of a man-powered aircraft, the knowledge of C_D at this lift coefficient allows us to make a very good approximation of the required power.

In Figure 13 a plot of $C_D/C_L^{3/2}$ versus C_L is presented for an aircraft having a wing area of 250 sq ft and an aspect ratio of 25. This aircraft was chosen as a typical representation of a very reasonable compromise for a man-powered two seater. When analyzing the sum of the lift drag and parasite drag coefficients divided by $C_L^{3/2}$, it becomes apparent that this sum has a very well defined minimum. In order to make the total power a minimum, the aerofoil characteristics have to be such as to have the lowest value of the profile drag divided by $C_L^{3/2}$ occurring right at the C_L value at which the minimum of the sum of the first two drag coefficients divided by $C_L^{3/2}$ is located.

There seems to be a very close relationship between these three drag coefficients and minimum power. There should always be an ideal aerofoil for each specific combination of wing area, aspect ratio and parasite drag. As stated previously, there is a comparatively meagre choice of aerofoil sections for which polars are available at the Reynolds numbers of interest to man-powered flight. As yet, no attempt has been made to design and/or test an aerofoil for such a specific application. From all the presently available data it is evident that not only the amount of camber but also its shape and proper thickness distribution should match the specific range of Reynolds number.

Nevertheless, the main value of the type of presentation shown in Figure 13 holds not only in facilitating the fitting of the best aerofoil to a chosen combination of wing configuration and parasite drag but in yielding something much more important, that is, a very clear insight into the pattern of the power distribution with varying C_L.

Firstly, let us consider a case of constant wing area and variable aspect ratio. The increase in aspect ratio will reduce the induced drag and will flatten the curve $C_{Di}/C_L^{3/2}$. After the effect of parasite drag has been added to it, the minimum of the sum would shift to the right, toward greater lift coefficients. This means that the best aerofoil for an aircraft having large aspect ratio wings has to have the minimum profile drag (divided by $C_L^{3/2}$) at higher C_L values, which in turn will require greater camber. This will undoubtedly increase the minimum profile drag and thereby partly reduce the beneficial effect of the increased aspect ratio.

When considering the opposite case, which would be the decreasing of the wing aspect ratio without changing the wing area, it would cause the shifting of the position of minimum drag to the left, toward smaller C_L values. This would, of course, result in a pronounced drag increase which would never be counterbalanced by the lower profile drag of an aerofoil especially designed for such a case.

In Figure 13, in addition to the NACA-6412 aerofoil section, for which all the basic calculations were

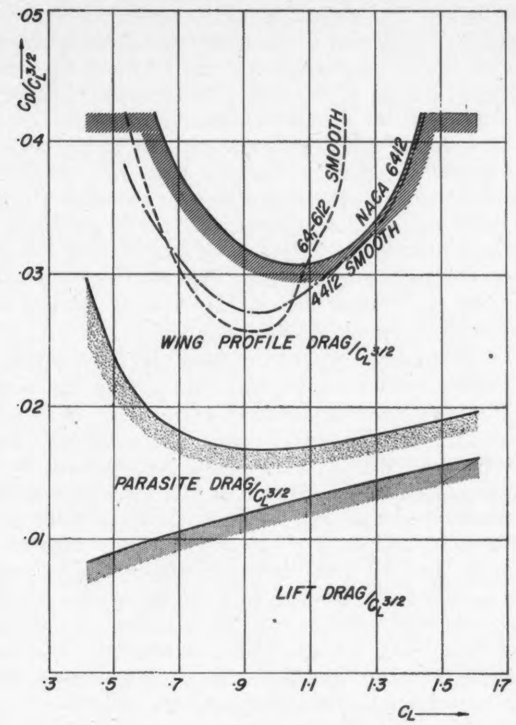

Figure 13

Drag breakdown for a man-powered two seater having a wing area of 250 sq ft and an aspect ratio of 25

made, two sections are shown with dotted lines. The first one is the laminar flow section NACA-64,-612 and the second the NACA-4412, both in the smooth surface condition. Both sections show very considerable reduction in power required as compared with the standard NACA-6412.

However, we do not know how much the results would improve if we had available the drag polar for the NACA-6412 in smooth surface condition, which would give us a fair comparison with the other two aerofoils. On the other hand we do not know, as has been mentioned before, how closely the smooth surface conditions could be reproduced on a full-scale aircraft wing. As a general conclusion it could be stated that without full-scale test results it would be impossible to make a reasonably honest appraisal of the performance of our man-powered aircraft.

WHERE DO WE GO FROM HERE?

The immediate outlook for a practical man-powered aircraft does not seem very good. There lies ahead of us a long trail of hard work and development, before any tangible improvements can be realized.

Reduce drag

As pointed out above, a serious attempt to reduce the drag has to be undertaken. Designers involved in the practical aspects of aircraft design usually cannot afford such scientific research or development. This type of work should be carried out by institutions such as the Royal Aircraft Establishment in Great Britain or the National Research Council in Canada, which possess all the necessary equipment and scientific personnel to pursue a research project of this type.

As the first major problem requiring solution, I would mention the problem of reducing the profile drag of a wing which would operate at Reynolds numbers ranging from 0.6 to 1.2×10^6 and fairly high lift coefficients. This is the Reynolds number range at which aerofoil sections usually show the greatest rate of change in profile drag and maximum lift. A combination of analytical investigation with wind tunnel and full-scale free air testing is recommended. Only by this method can the results be directly applicable to full-scale aeroplanes. The greatest difficulty may derive from the impossibility of maintaining the geometrical shape of the wing accurately enough to assure the lowest profile drag. Therefore, it would be advisable to use, in the free air testing, a portion of the full-scale wing most realistically representing the wing structure and exterior finish.

Transmission efficiency

It is evident from Figure 14 that better power consumption could be achieved by improving the mechanical efficiency of the transmission and the propulsive efficiency of the propellers.

Perhaps the most promising of all fields for improvement is mechanical transmission. The long distance from the foot pedals or hand cranks to the propeller shaft or undercarriage wheels requires quite a number of gear trains, driving parallel shaft systems and at least one changing the torque vector through approximately 90°. Positive transmissions (chains and gears) as well as friction belts or ropes have been used on man-powered aircraft. From the theory of machine elements there are a number of general recommendations available on how to lay out and design a transmission in order to achieve the highest possible mechanical efficiency.

The angular change of the torque vector, which is usually achieved by a pair of bevel gears or a twisted belt, offers plenty of scope for improvement, and the author believes that it should not be difficult to devise a few practical schemes which would raise the total efficiency of the transmission up to 94 to 96%.

Of course, in order to obtain optimum efficiency, the power input has to be fed into the transmission system at the highest torque/rpm ratio, when using the optimum foot pedal or crank radius. The same statement applies to the output end, where the mechanical conditions of producing power should match the optimum aerodynamics of the propellers. Any improvement in total propulsion efficiency should be set as an important goal for a designer.

Propeller efficiency

In the field of propeller design especially, more theoretical and experimental work has to be done at the scientific institution level. The problem of propeller analysis and design for this specific speed range and application is so important for the final success of a man-powered aircraft that it deserves separate

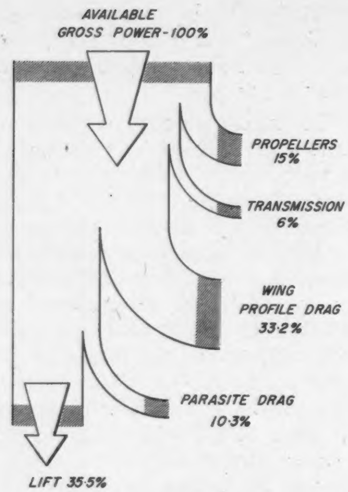

Figure 14
Gross power consumption distribution in the case of a man-powered two seater having a wing area of 250 sq ft and an aspect ratio of 25

attention and should be covered in a series of reports and articles.

Regarding the technological aspects of building low power, large diameter propellers, the best blades would be manufactured from a combination of plastic foam and fibreglass fabric. The propeller hub should provide for variable pitch, in order to obtain maximum propeller efficiency in powered flight, as well as feathering during soaring. Variable pitch propellers can even be used as very efficient drag-increasing devices to serve as air-brakes during descent and landing.

Miscellaneous features

There still exist quite a number of items or hardware parts which should be developed to yield the best performance – for example, undercarriage wheels with specially light tires, seats, flight control and transmission components, cockpit and fuselage design, etc. All these features present a challenge to the designer in his attempts to save weight. Probably the most important part of the design work will be the general layout, relating the aircraft parts in such a manner as to produce an inherently light structure.

A successful man-powered aircraft is a marginal possibility, requiring some noticeable improvements in almost every item or parameter entering the problem. The total sum of all improvements may become sufficient to tip the balance in favour of practical reality.

CONCLUSION

What could be designed and built today is an aircraft which would, by man-power alone, be able to perform flights of a few minutes duration at the most. Flying within the ground cushion may prolong the time considerably and therefore the design of such an aircraft should take into consideration the full utilization of this phenomenon.

More and more full-scale aircraft should be built, as each one will bring new ideas and experience and will hasten the realization of successful man-power flight. Probably this is the most appropriate place to express our full appreciation of the efforts being made in Great Britain in this field. Two main events come to mind; one the Kremer Competition and the other the creation of a Man Powered Aircraft Group within the Royal Aeronautical Society, ready to give financial assistance to prospective designers in their efforts to build aircraft. Let us hope that the Kremer Competition and the Man Powered Aircraft Group will, in time, develop something much more serious and permanent, as Ursinus did in the field of gliding in Germany in the early twenties.

If fully successful, aerocycling may develop into something even more popular than gliding and soaring. A man-powered aircraft being a very clean and light aircraft will possess better soaring ability than any sailplane. Being an aircraft of exceptionally small sinking speed it will be able to take advantage of all the weak thermals and slope-winds which could not be used by sailplanes. This potential advantage may help to develop man-powered flight into a popular and interesting sport which could be practised anywhere, would combine all the pleasures of cycling and flying and, for the first time, would blend the skills of the pilot and the athlete. Undoubtedly this can be achieved, if we recognize the difficulties and are prepared to adopt a sober and scientific approach to their solution.

REFERENCES

(1) Ursinus, O. – *Mitteilungen der Muskelflug Institutes*, FLUGSPORT, 1936 TO 1937.

(2) Hoerner, S. F. – *Fluid Dynamic Drag*, 1958.

(3) *Airfoil Section Characteristics as Affected by Variation of the Reynolds Number*, NACA REPORT 586.

(4) *Aerodynamic Characteristics of 15 NACA Aerofoil Sections at 7 Reynolds Numbers*, NACA TECH. NOTE 1945.

(5) GROUND EFFECT SYMPOSIUM, PRINCETON UNIVERSITY, 1959.

(6) Raspet, A. – *The Phoenix as a Solution to Optimum Cross Country Soaring*, AERO REVIEW, JUNE, 1960.

BIBLIOGRAPHY

(1) Shenstone, B. S. – *The Problem of the Very Lightweight, Highly Efficient Aeroplane*, CANADIAN AERONAUTICAL JOURNAL, MARCH, 1956.

(2) Shenstone, B. S. – *Engineering Aspects in Man-powered Flight*, JOURNAL OF THE RAeS, AUGUST, 1960.

(3) Nonweiler, T. – *The Man-powered Aircraft – A Design Study*, JOURNAL OF THE RAeS, OCTOBER, 1958.

(4) Wilkie, D. R. – *Man as an Aero Engine*, JOURNAL OF THE RAeS, AUGUST, 1960.

(5) BRITISH CIVIL AIRWORTHINESS REQUIREMENTS, SECTION E – GLIDERS.

(6) *Design of Wooden Aircraft Structures*, ANC-18.

(7) *Das Haessler-Villinger Muskelkraft Flugzeug*, FLUGSPORT, 28TH APRIL, 1937.

(8) *Muskel Flugzeug Bossi*, AER CYCLE, FLUGSPORT, 14TH APRIL, 1937.

(9) *Muskelkraft Flugzeug Bossi-Bonomi*, FLUGSPORT, 28TH APRIL, 1937.

(10) Bossi, E. – *A Man Has Flown by his Own Power in 1937*, CANADIAN AERONAUTICAL JOURNAL, DECEMBER, 1960.

(11) Morelli, P. – *Weight of Sailplanes as a Function of Their Main Geometric Parameters* (in Polish), TECHNIKA LOTMICZA, JANUARY, 1960.

THE CALCULATION OF THERMAL STRESSES IN IMPELLERS WITH LATERAL VANES†

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SUMMARY

The present paper describes a method of computing the stresses due to a radial temperature gradient in a disk of given profile with lateral vanes. The vanes are considered as carrying their share of the radial stress. The temperature distribution is assumed to be represented with sufficient accuracy by a polynomial of arbitrary degree.

LIST OF SYMBOLS

A, B, C	constants, defined by Eq. (30)
D	defined by Eq. (56)
E	Young's modulus (lb/in^2)
G_i	temperature constants
K	integration constant
M_i, N_i	defined by Eq. (36)
P, Q	defined by Eq. (53)
S	defined by Eq. (56)
T	temperature difference (°F)
V	area of one vane
a, b, c	constants, defined by Eq. (16)
a', b', c'	constants, defined by Eq. (16)
h	a constant
k	a variable, defined by Eq. (22)
m	correction factor, defined by Eq. (62)
q	ratio of vane area to disk area
r	radius (inches)
s	number of vanes
u	radial expansion (inches)
z	$k\sigma_r + \sigma_\theta$
α	linear thermal expansion coefficient (1/°F)
$\epsilon_r, \epsilon_\theta$	radial, tangential strain
μ	exponent in profile equation
ν	Poisson's ratio
σ_r	radial stress in disk (lb/in^2)
σ_v	radial stress in vanes (lb/in^2)
σ_θ	tangential stress in disk (lb/in^2)
ζ	Defined by Eq. (55)
ξ	Defined by Eq. (55)

INTRODUCTION

THE PURPOSE of this paper is to present a method for the determination of stresses due to a radial temperature gradient in an impeller of given profile and given arrangement of lateral vanes. The calculation is a step by step procedure. The solid disk part of the impeller is approximated by a constant hyperbolic profile for each step. The total cross-sectional area of all vanes is approximated by a constant area for each step. The vanes are subjected to the same temperature as the disk and carry their share of the radial stress. The temperature distribution is expressed in the form of a polynomial of arbitrary degree, and thus can be made to represent the actual temperature to any degree of accuracy desired.

The method outlined is an application of work by Tumarkin[1, 2] and Reeman and Gray[3] to the temperature stress problem.

FUNDAMENTAL EQUATIONS

Apart from the assumption that the material is elastic, it is assumed that the disk is so thin that the longitudinal stress is zero. The strains are assumed not to vary along the longitudinal direction of the disk. An inspection of the radial equilibrium of an element of the disk, as shown in Figure 1, will lead to the equilibrium equation

$$\frac{d(ry\sigma_r)}{dr} + \frac{d}{dr}\left(\frac{sV\sigma_v}{2\pi}\right) - \sigma_\theta y = 0 \tag{1}$$

If we let the total radial strain be ϵ_r, the total tangential strain be ϵ_θ and the linear strain due to the temperature gradient T be αT, and if we recall that the longitudinal stress is zero, we may establish the relation between radial and tangential strains and radial and tangential stresses for the disk as

$$\epsilon_r - \alpha T = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) \tag{2}$$

and

$$\epsilon_\theta - \alpha T = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) \tag{3}$$

For the vanes, which cannot sustain any tangential stress and which are assumed to have the same radial strain at a given radius as the disk, we may write

$$\epsilon_r - \alpha T = \frac{1}{E}\sigma_v \tag{4}$$

A comparison of Eqs. (2) and (4) will show that the radial stress σ_v in the vanes is related to the radial and tangential stresses in the disk by

$$\sigma_v = \sigma_r - \nu\sigma_\theta \tag{5}$$

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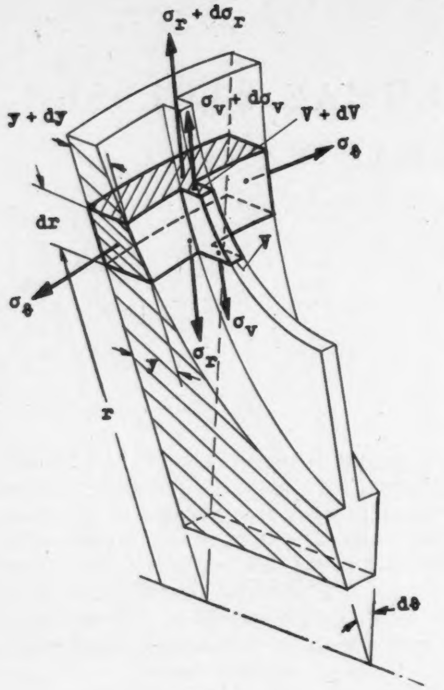

Figure 1
Element of impeller

By substituting the value of σ_r, as given by Eq. (5), in Eq. (1), the following expression for the equilibrium equation is obtained

$$\frac{d(ry\sigma_r)}{dr} + \frac{d}{dr}\left(\frac{sV\sigma_r}{2\pi}\right) - \frac{d}{dr}\left(\frac{sV\nu\sigma_\theta}{2\pi}\right) - \sigma_\theta y = 0 \tag{6}$$

To facilitate further analysis, we now make the assumption that the total cross-sectional area sV of the vanes at a radius r is related to the cross-sectional area $2\pi ry$ of the disk at the same radius by a factor q, which is considered constant for each step in the calculation.

$$sV = q \cdot 2\pi ry \tag{7}$$

After substituting $sV = q \cdot 2\pi ry$ in Eq. (6), we have

$$(1 + q)\frac{d(ry\sigma_r)}{dr} - \frac{d(\nu qry\sigma_\theta)}{dr} - \sigma_\theta y = 0 \tag{8}$$

Execution of the differentiations in Eq. (8) and the subsequent division by ry, gives

$$(1 + q)\frac{d\sigma_r}{dr} + (1 + q)\left[\frac{1}{r} + \frac{d \ln y}{dr}\right]\sigma_r - \nu q\frac{d\sigma_\theta}{dr} - \left[\frac{1}{r} + \nu q\left(\frac{1}{r} + \frac{d\ln y}{dr}\right)\right]\sigma_\theta = 0 \tag{9}$$

The total radial and tangential strains can be shown[4] to be related to the radial deformation u by

$$\epsilon_r = \frac{du}{dr} \quad \text{and} \quad \epsilon_\theta = \frac{u}{r} \tag{10}$$

These expressions may be entered into Eqs. (2) and (3), which then become

$$\frac{du}{dr} = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) + \alpha T \quad \text{and} \quad \frac{u}{r} = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) + \alpha T \tag{11}$$

Differentiation of the second expression of Eq. (11) with respect to r, entering the result into the first equation, and rearranging

$$\frac{d\sigma_\theta}{dr} = \frac{1 + \nu}{r}(\sigma_r - \sigma_\theta) + \nu\frac{d\sigma_r}{dr} - \alpha E\frac{dT}{dr} \tag{12}$$

If one now replaces $d\sigma_\theta/dr$ in Eq. (9) by the expression of Eq. (12), one may write

$$(1 + q - \nu^2q)\frac{d\sigma_r}{dr} + \left(\frac{1 + q + \nu q - \nu^2 q}{r} + (1 + q)\frac{d\ln y}{dr}\right)\sigma_r + \left(\frac{\nu^2q - 1}{r} - \nu q\frac{d\ln y}{dr}\right)\sigma_\theta + \nu q\,\alpha E\frac{dT}{dr} = 0 \tag{13}$$

It follows from Eq. (13) that

$$\frac{d\sigma_r}{dr} = -\left[\frac{1 + q - \nu q - \nu^2q}{1 + q - \nu^2q}\,\frac{1}{r} + \frac{1 + q}{1 + q - \nu^2q}\,\frac{d\ln y}{dr}\right]\sigma_r + \left[\frac{1 - \nu^2q}{1 + q - \nu^2q}\,\frac{1}{r} + \frac{\nu q}{1 + q - \nu^2q}\,\frac{d\ln y}{dr}\right]\sigma_\theta - \frac{\nu q}{1 + q - \nu^2q}\,\alpha E\frac{dT}{dr} \tag{14}$$

Substituting the expression for $d\sigma_r/dr$ from Eq. (14) in Eq. (12) results in

$$\frac{d\sigma_\theta}{dr} = \left[\frac{1 + q}{1 + q - \nu^2q}\,\frac{1}{r} - \frac{\nu(1 + q)}{1 + q - \nu^2q}\,\frac{d\ln y}{dr}\right]\sigma_r - \left[\frac{1 + q + \nu q - \nu^2q}{1 + q - \nu^2q}\,\frac{1}{r} - \frac{\nu^2q}{1 + q - \nu^2q}\,\frac{d\ln y}{dr}\right]\sigma_\theta - \frac{1 + q}{1 + q - \nu^2q}\,\alpha E\frac{dT}{dr} \tag{15}$$

Let us now shorten Eqs. (14) and (15) to

$$\frac{d\sigma_r}{dr} = a\sigma_r + b\sigma_\theta + c \quad \text{and} \quad \frac{d\sigma_\theta}{dr} = a'\sigma_r + b'\sigma_\theta + c' \tag{16}$$

where a, b, c, a', b' and c' are obtained by comparing the terms in Eq. (16) with the corresponding terms in Eqs. (14) and (15). The system of two expressions in Eq. (16) may be reduced to one differential equation, if we introduce a new variable

$$z = k\sigma_r + \sigma_\theta \tag{17}$$

Differentiating Eq. (17) with respect to r,

$$\frac{dz}{dr} = \frac{dk}{dr}\sigma_r + \frac{d\sigma_r}{dr}k + \frac{d\sigma_\theta}{dr} \tag{18}$$

Introducing $d\sigma_r/dr$ and $d\sigma_\theta/dr$ from Eq. (16), Eq. (18) becomes

$$\frac{dz}{dr} = \left(\frac{dk}{dr} + ak + a'\right)\sigma_r + (bk + b')\sigma_\theta + (ck + c') \tag{19}$$

The value of k may be chosen in such manner that the right hand side of Eq. (19) depends on z only, without containing σ_r and σ_θ explicitly. This is achieved by making the ratio of the coefficients which precede σ_r and σ_θ in Eq. (19) equal to the ratio of those in the equation $z = k\sigma_r + \sigma_\theta$, hence

$$\frac{\dfrac{dk}{dr} + ak + a'}{bk + b'} = \frac{k}{1} \tag{20}$$

Eq. (19) consequently becomes

$$\frac{dz}{dr} = (bk + b')z + (ck + c') \tag{21}$$

and thus the desired single linear differential equation is obtained.

Eq. (20) may be rewritten for the determination of k in form of a Riccati differential equation

$$\frac{dk}{dr} = bk^2 + (b' - a)k - a' \tag{22}$$

Reintroduction of the values for b, b', a and a', as obtained from Eqs. (14), (15) and (16), gives then for Eq. (22),

$$\frac{dk}{dr} = \left[\frac{1 - \nu^2 q}{1 + q - \nu^2 q}\,\frac{1}{r} + \frac{\nu q}{1 + q - \nu^2 q}\,\frac{d \ln y}{dr}\right]k^2 + \left[\frac{-2\nu q}{1 + q - \nu^2 q}\,\frac{1}{r} + \frac{1 + q + \nu^2 q}{1 + q - \nu^2 q}\,\frac{d \ln y}{dr}\right]k - \left[\frac{1 + q}{1 + q - \nu^2 q}\,\frac{1}{r} - \frac{\nu(1 + q)}{1 + q - \nu^2 q}\,\frac{d \ln y}{dr}\right] \tag{23}$$

Let us, at this point, introduce the mathematical expression for the thickness y of the disk,

$$y = hr^{-\mu} \tag{24}$$

where μ is a constant over each step. With y as given by Eq. (24), Eq. (23) becomes

$$\frac{dk}{dr} = \frac{1}{1 + q - \nu^2 q}\,\frac{1}{r}\Big[[(1 - \nu q(\nu + \mu))]k^2 - [(1 + q + \nu^2 q)\mu + 2\nu q]k - (1 + q)(1 + \nu\mu)\Big] \tag{25}$$

This differential equation is fulfilled for $k =$ const, whence $dk/dr = 0$, or

$$[1 - \nu q(\nu + \mu)]k^2 - [(1 + q + \nu^2 q)\mu + 2\nu q]k - (1 + \nu\mu)(1 + q) = 0 \tag{26}$$

from which we obtain two values k_1 and k_2 for k. They are

$$k_{1,2} = \frac{[(1+q+\nu^2 q)\mu + 2\nu q] \pm \sqrt{(1+q-\nu^2 q)[(1+q-\nu^2 q)\mu^2 + 4\nu\mu + 4]}}{2[1 - \nu q(\nu + \mu)]} \tag{27}$$

With values k_1 and k_2 known, we can now solve the differential Eq. (21) for the variable z. Replacing b, b', c and c' by their original expressions, Eq. (21) reads

$$\frac{dz}{dr} = \left[\left(\frac{1 - \nu^2 q}{1 + q - \nu^2 q}\,\frac{1}{r} + \frac{\nu q}{1 + q - \nu^2 q}\,\frac{d \ln y}{dr}\right)k - \left(\frac{1 + q + \nu q - \nu^2 q}{1 + q - \nu^2 q}\,\frac{1}{r} - \frac{\nu^2 q}{1 + q - \nu^2 q}\,\frac{d \ln y}{dr}\right)\right]z - \frac{\nu q k + 1 + q}{1 + q - \nu^2 q}\,\alpha E\,\frac{dT}{dr} \tag{28}$$

Recalling now that we have introduced the expression of Eq. (24) for the thickness y of the disk, we may write Eq. (28) in the form

$$(1 + q - \nu^2 q)\frac{dz}{dr} = \left[\frac{1 - \nu^2 q - \nu\mu q}{r}\,k - \frac{1 + q + \nu q + (\mu - 1)\nu^2 q}{r}\right]z - (\nu q k + 1 + q)\,\alpha E\,\frac{dT}{dr} \tag{29}$$

For ease of handling let us introduce three quantities A, B and C and write Eq. (29) as

$$A\,\frac{dz}{dr} = \frac{Bk - C}{r}\,z - (\nu q k + 1 + q)\,\alpha E\,\frac{dT}{dr} \tag{30}$$

We may now solve this expression for z and obtain

$$z = k\sigma_r + \sigma_\theta = \exp\int\left(\frac{B}{A}k - \frac{C}{A}\right)\frac{dr}{r}\cdot\left[K - \frac{\nu q k + 1 + q}{A}\,\alpha E \cdot \int\frac{dT}{dr}\exp\left(-\int\left(\frac{B}{A}k - \frac{C}{A}\right)\frac{dr}{r}\right)dr\right] \tag{31}$$

where K is the constant of integration.
The integration can be carried out partially, whence

$$z = r^{Bk/A - C/A}\left[K - \frac{\nu q k + 1 + q}{A}\,\alpha E\int\frac{dT}{dr}\,r^{-(Bk/A - C/A)}\,dr\right] \tag{32}$$

We must now specify in which way the temperature difference T is dependent upon the radius r. We shall assume that T can adequately be represented by a polynomial of the form

$$T = G_i r^i \tag{33}$$

where $i = 0, 1, 2, 3, 4, \ldots . p$. The summation convention is to apply, i.e.

$$G_i r^i \equiv \sum_{i=0}^{p} G_i r^i$$

Differentiation of Eq. (33) with respect to r gives

$$\frac{dT}{dr} = i\,G_i r^{i-1} \tag{34}$$

Entering the expression for dT/dr of Eq. (34) into Eq. (32) and integrating,

$$z = k\sigma_r + \sigma_\theta = r^{Bk/A - C/A}\left[K - \frac{i\,G_i\;\alpha E(\nu q k + 1 + q)}{A(i - Bk/A + C/A)}\cdot r^{i - Bk/A + C/A}\right] \tag{35}$$

Recalling that there are two values for k, and setting

$$\left.\begin{aligned} M_1 &= \frac{G_1\, \alpha E(\nu q k_1 + 1 + q)}{A} \frac{i}{i - \frac{B}{A}k_1 + \frac{C}{A}} \\ N_1 &= \frac{G_1\, \alpha E(\nu q k_2 + 1 + q)}{A} \frac{i}{i - \frac{B}{A}k_2 + \frac{C}{A}} \end{aligned}\right\} \quad (36)$$

we obtain finally for the relation between stresses, by solving Eq. (35) for K,

$$\left.\begin{aligned} \frac{k_1\sigma_r + \sigma_\theta + M_1 r^i}{r^{Bk_1/A - C/A}} &= \text{const} \\ \frac{k_2\sigma_r + \sigma_\theta + N_1 r^i}{r^{Bk_2/A - C/A}} &= \text{const} \end{aligned}\right\} \quad (37)$$

DISCONTINUITIES

If there is a discontinuity in the thickness of the vanes and/or the disk (Figure 2), there is a corresponding discontinuity in the stress.

Let the stresses on the one side of the discontinuity be σ_r', σ_θ' and σ_v', and those on the other side σ_r'', σ_θ'' and σ_v''. Equilibrium requires, for the assumption that each stress is uniformly distributed over a cylindrical section of the impeller, that

$$\sigma_r'y' + \sigma_v'q'y' = \sigma_r''y'' + \sigma_v''q''y'' \quad (38)$$

Applying Eq. (5), Eq. (38) can be changed to read

$$\sigma_r'y' + (\sigma_r' - \nu\sigma_\theta')q'y' = \sigma_r''y'' + (\sigma_r'' - \nu\sigma_\theta'')q''y'' \quad (39)$$

The strains are continuous across a discontinuity of wall thickness, hence $\epsilon_\theta' = \epsilon_\theta''$, which, in view of Eq. (3), leads to

$$\sigma_\theta' - \nu\sigma_r' = \sigma_\theta'' - \nu\sigma_r'' \quad (40)$$

whence

$$\sigma_\theta'' = \sigma_\theta' + \nu(\sigma_r'' - \sigma_r') \quad (41)$$

Substituting for σ_θ'' in Eq. (39)

$$\sigma_r'' = \frac{y'}{y''} \frac{\sigma_r'(1 + q' - \nu^2 q''y''/y') + \nu\sigma_\theta'(q''y''/y' - q')}{1 + q'' - \nu^2 q''} \quad (42)$$

Eq. (42) is used to determine the radial stress σ_r''. The tangential stress σ_θ'' beyond the discontinuity is found, in turn, by entering the value of σ_r'' into Eq. (41).

Eqs. (41) and (42) were derived for a discontinuity in disk and vanes. There may be a discontinuity in the vanes at a certain section, without a discontinuity in the disk at the same section. In this case $y'' = y'$, and Eq. (42) is simplified to

$$\sigma_r'' = \frac{\sigma_r'(1 + q' - \nu^2 q'') + \nu\sigma_\theta'(q'' - q')}{1 + q'' - \nu^2 q''} \quad (43)$$

Frequently there is a discontinuity in the disk at a certain section, and the vanes are continuous at the same section. For this case $q' = q''\, y''/y'$, and Eq. (42) becomes

$$\sigma_r'' = \frac{y'}{y''} \frac{\sigma_r'(1 + (1 - \nu^2)q')}{1 + (1 - \nu^2)q'y'/y''} \quad (44)$$

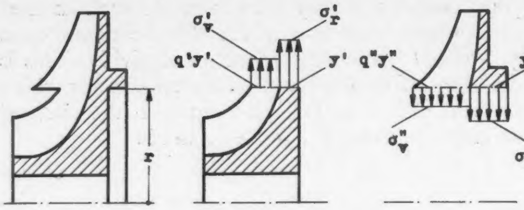

Figure 2
Discontinuities

SPECIAL CASES

It may happen, that the root k_1 has a value which makes one of the integers i equal to

$$i = \frac{B}{A}k_1 - \frac{C}{A} \quad (45)$$

As a consequence of this, a logarithmic term appears in the solution of the integral in Eq. (32). The first expression of Eq. (37) must therefore be replaced by

$$\frac{k_1\sigma_r + \sigma_\theta + M_j r^j + (\nu q k_1 + 1 + q)\frac{\alpha E}{A} i\, G_i r^i \ln r}{r^i} = \text{const} \quad (46)$$

where i is equal to the value given by Eq. (45) and $j = 1, 2, \ldots, (i-1), (i+1), \ldots, p$.

Another special case arises when

$$1 - \nu q(\nu + \mu) = 0 \quad (47)$$

Then, from Eq. (27),

$$k_1 = \infty \quad (48)$$

and the first expression of Eq. (37) becomes

$$\frac{\sigma_r + M_1 r^i/k_1}{r^{Bk_1/A - C/A}} = \text{const} \quad (49)$$

From Eqs. (27), (29) and (30), we find that

$$\frac{B}{A}k_1 - \frac{C}{A} = \frac{\mu}{2} - 1 + \sqrt{\frac{\mu^2}{2} + \frac{1 + \nu\mu}{1 + q - \nu^2 q}} \quad (50)$$

which is generally true. It is easily seen that the expression $Bk_1/A - C/A$ is finite when $k_1 = \infty$. The expression M_1/k_1 in Eq. (49) is also finite for $k_1 = \infty$, as an inspection of the first expression of Eq. (36) will indicate. The application of the Bernoulli-L'Hospital rule (by using μ for example as the variable) to the second expression of Eq. (27) gives after simplication

$$k_2 = -\frac{(1 + q)(1 + \nu\mu)}{(1 + q + \nu^2 q)\mu + 2\nu q} \quad (51)$$

for the condition expressed by Eq. (47).

CALCULATION PROCEDURES

(1) The impeller is divided into annular sections by working off various radii r_n. Each discontinuity must be marked off by a radius.

(2) The thickness y of the disk (without vanes) is measured at these radii and the average exponent μ for each section determined from

$$\mu = \frac{\lg y_n - \lg y_{n+1}}{\lg r_{n+1} - \lg r_n} \tag{52}$$

Eq. (52) is obtained by rewriting Eq. (24) for the inner and outer radius of each annular section, and eliminating the term lg h.

(3) The total area sV of the vanes is measured at each radius and the value of the quantity q is determined by means of Eq. (7). Then an average value for q is calculated for each section.

(4) The values for k_1 and k_2 are then computed by means of Eq. (27).

(5) By using Eqs. (27), (29) and (30), we can find

$$\left.\begin{aligned} \frac{B}{A}k_1 - \frac{C}{A} &= \frac{\mu}{2} - 1 + \sqrt{\frac{\mu^2}{4} + \frac{1+\nu\mu}{1+q-\nu^2 q}} \\ \frac{B}{A}k_2 - \frac{C}{A} &= \frac{\mu}{2} - 1 - \sqrt{\frac{\mu^2}{4} + \frac{1+\nu\mu}{1+q-\nu^2 q}} \end{aligned}\right\} \tag{53}$$

The numerical values found for the expressions of Eq. (53) for each annular section, are subsequently used to form the expressions

$$\begin{aligned} P &= \left(\frac{r_{n+1}}{r_n}\right)^{Bk_1/A-C/A} \\ Q &= \left(\frac{r_{n+1}}{r_n}\right)^{Bk_2/A-C/A} \end{aligned} \tag{54}$$

where r_n and r_{n+1} are the inner and outer radii bounding each section.

(6) M_i and N_i, given by Eq. (36), are calculated for each section and the expressions

$$\begin{aligned} \zeta &= M_i r^i \\ \xi &= N_i r^i \end{aligned} \tag{55}$$

formed for each radius. (The summation convention applies.) Note that, at each radius, two values for ζ as well as ξ are found, one value to be used for the section bounded by on the outside by the radius, the other value to be used for the section bounded at the inside by the radius.

(7) Let Eq. (37) be written

$$\begin{aligned} \frac{S+\zeta}{r^{(Bk_1-C)/A}} &= \text{const} \\ \frac{D+\xi}{r^{(Bk_2-C)/A}} &= \text{const} \end{aligned} \tag{56}$$

where

$$\begin{aligned} S &= k_1\sigma_r + \sigma_\theta \\ D &= k_2\sigma_r + \sigma_\theta \end{aligned} \tag{57}$$

Whence, by using Eq. (54), one obtains for the transition from the inner to the outer radius of each section

$$\begin{aligned} S_{n+1} &= P(S_n + \zeta_n) - \zeta_{n+1} \\ D_{n+1} &= Q(D_n + \xi_n) - \xi_{n+1} \end{aligned} \tag{58}$$

With S and D known at each radius, one can calculate the actual stresses from Eq. (57) as

$$\begin{aligned} \sigma_r &= \frac{S-D}{k_1-k_2} \\ \sigma_\theta &= D - k_2\sigma_r \end{aligned} \tag{59}$$

(8) If the impeller is solid, the computation is begun at $r = 0$, where $\sigma_\theta = \sigma_r$. A value for σ_r is assumed and S_o and D_o are determined by means of Eq. (57). Eqs. (58) and (59) are then employed and the stresses at $r = r_1$ determined. The procedure is repeated until the outside radius is reached.

For an impeller with bore, the computation is begun at the bore radius $r = r_o$. At the bore, the radial stress is zero. We begin therefore with $\sigma_r = 0$ and an arbitrary value for σ_θ. The subsequent calculations are carried out as described above.

(9) Since in either case the radial stress calculated for the outside radius will generally not be found to be equal to zero, as the boundary condition requires, we have to carry out a second stress computation. The computation is again begun by choosing an arbitrary $\sigma_r = \sigma_\theta$ at the centre of the solid disk, or $\sigma_r = 0$ and an arbitrary σ_θ at the bore of the hollow disk; but in this calculation the impeller is free from temperature influences and consequently ζ and ξ will be zero and Eqs. (56) and (58) be modified accordingly.

(10) In order to obtain the actual stresses σ_r and σ_θ in the impeller, the stresses $(\sigma_r)_{II}$ and $(\sigma_\theta)_{II}$ obtained in the second computation are multiplied by a factor m, and then added to the stresses $(\sigma_r)_I$ and $(\sigma_\theta)_I$ obtained in the first computation. Hence the actual stresses at each radius are

$$\begin{aligned} \sigma_r &= (\sigma_r)_I + m(\sigma_r)_{II} \\ \sigma_\theta &= (\sigma_\theta)_I + m(\sigma_\theta)_{II} \end{aligned} \tag{60}$$

The factor m is constant for the whole impeller and is found from the condition that the total radial force (acting on disk and vanes) is zero at the outside radius.

$$\sigma_r + q\sigma_v = 0 \tag{61}$$

Applying Eq. (5), Eq. (61) may be written

$$(1+q)\sigma_r - \nu q\sigma_\theta = 0 \tag{62}$$

Entering σ_r and σ_θ from Eq. (60) into Eq. (62) and rearranging,

$$m = -\frac{(1+q)(\sigma_r)_I - \nu q(\sigma_\theta)_I}{(1+q)(\sigma_r)_{II} - \nu q(\sigma_\theta)_{II}} \tag{63}$$

The stresses appearing in Eq. (63) are those at the outside radius, and the quantity q is that for the outside annular section.

(11) Having obtained σ_r and σ_θ throughout the impeller, the stress σ_v in the vanes may be computed for each radius by means of Eq. (5).

(12) At discontinuities, or when one or both of the special cases described previously occur, the computation procedure must be modified as described under "Discontinuities" and "Special Cases".

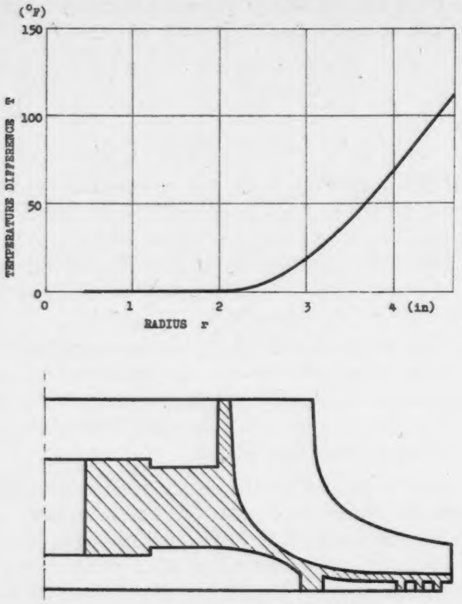

Figure 3
Impeller profile and temperature distribution

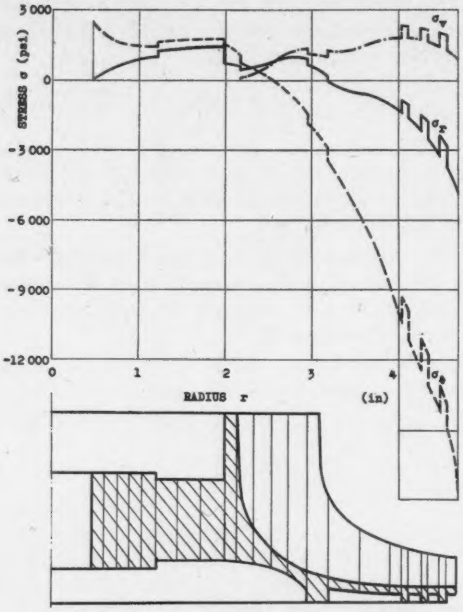

Figure 4
Thermal stresses in impeller

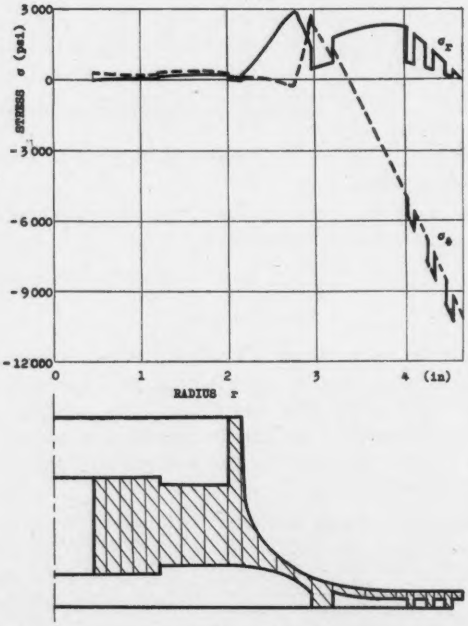

Figure 5
Thermal stresses in impeller disk without vanes

Example

Given is an impeller with vanes as shown in Figure 3. The temperature difference is zero at the bore, and distributed radially according to

$$T = 9.43r - 8.21r^2 - 0.11r^3 + 1.42r^4 - 0.187r^5$$

The temperature distribution is also indicated in Figure 3.

The radial and tangential stresses, σ_r and σ_θ, and the radial stress σ_v in the vanes, were computed and are plotted in Figure 4. For a comparison, the stresses in a disk of the same dimensions but without lateral vanes, and subjected to the same temperature distribution, were computed, and are shown in Figure 5.

CONCLUSION

A method is presented for the determination of thermal stresses in an impeller with lateral vanes, based on the assumptions that the radial strains do not vary longitudinally and that the disk profile can be represented with sufficient accuracy by a hyperbola for each section. The computation is accomplished by dividing the impeller into a number of annular sections. The computation procedure is described in detail. The results of the stress computation are presented as an example. The addition of lateral vanes to a disk is found to lead to larger (absolute) thermal stresses.

ACKNOWLEDGMENTS

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REFERENCES

(1) Tumarkin, S. – *Methods of Stress Calculation in Rotating Disks*, Report No. 262, Central Aero-Hydrodynamical Institute, Moscow, 1936.

(2) Tumarkin, S. – *Methods of Stress Calculations in Rotating Disks*, (English translation by S. Reiss), NACA Technical Memorandum No. 1064, Washington, 1944.

(3) Reeman, J., and Gray, P. G. – *The Calculation of Centrifugal Stresses in Unshrouded Impellers*, R.A.E. Technical Note No. Eng 238.

(4) Timoshenko, S. – *Strength of Materials, Part II*, D. van Nostrand Co., p 206, Toronto, 1956.

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RAL VANES

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TECHNICAL FORUM

Runway Ice Removal†

BY W/C W. N. HOYE, M.C.A.I.

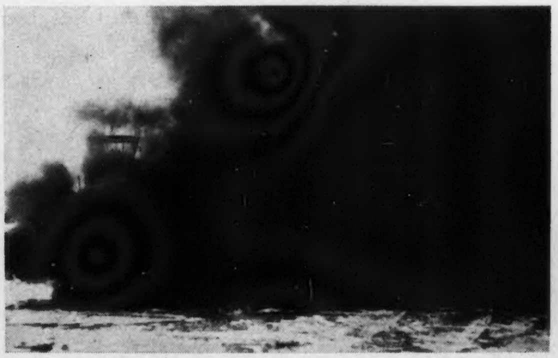
Royal Canadian Air Force

IT sometimes happens that a runway is rendered totally unserviceable by a combination of circumstances and icing conditions which render normal remedies ineffectual. This report describes a method employed in such an emergency with good results.

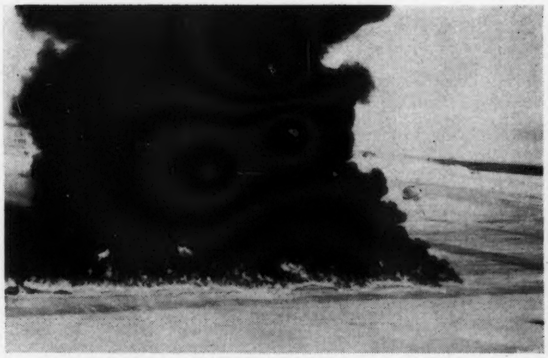
At an RCAF station equipped with the usual provisions for snow removal and ice control, freezing rain had deposited up to a ½ inch of clear ice on runways that had been swept bare, resulting in a smooth and very slippery surface; this became extremely hard when the temperature dropped to –20°F overnight, with a 10 to 15 mph wind.

Normal sanding was ineffective because the sand was blown off by the wind as quickly as it was applied. The normal stock pile of heated sand proved completely inadequate because it was not hot enough to stick to the surface. Perforated pipes were inserted in loaded sand trucks and connected to steam lines to heat the sand and truck body. This raised the temperature considerably, but the chill factor was so great that sand was still cold by the time it was through the spreader and on to the ice.

Scarifying and breaking up the ice by mechanical means was also tried without success. Heat was then applied to the surface from the exhaust of a T33 aircraft. This was accomplished by lowering the main oleos and extending the nose oleo to change the aircraft attitude. Taxiing the aircraft resulted in some patchy clearance of ice from areas 10 to 15 ft wide. However, this method was not very successful because large throttle openings (over 55%) were necessary and the aircraft then became uncontrollable on the slippery surface.



Removal of runway ice by burning jet fuel



Some idea of the scale may be obtained from the fuel truck in the background

Direct application of heat to the surface was then tried by spilling jet fuel on the ice and burning the fuel. This proved quite effective as had been expected, and the main concern was the effect the heat might have on the tarmac. However, under the conditions which existed, the surface was still cold (estimated 40° to 50°F) when the flames subsided. There was no apparent effect on the concrete or filler strips.

The method used was to spread fuel in double strips by men walking along holding open the nozzles of the refuelling tender hoses. Lighting was quite difficult and eventually a portable asphalt torch was used to light the fuel all along the strip. Ice under the fuel melted and the water flowed down towards the edge of the tarmac carrying burning fuel on top of it. In this way nearly all the surface water drained off the burned areas. Sanding was carried out immediately after the flames subsided and was effectively frozen into any remaining surface water. The result was a strip of 50 to 75 ft wide with good braking traction. Since the burning did not affect the asphalt strips in the tarmac the same procedure was carried out on taxi strips and runways, and proved to be equally effective without damage to the surface. The Figures illustrate the process used.

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The Applied Scientist — A New Member of the Engineering Team

ENGINEERING METAPHYSICS

The education of tomorrow's engineers is under study by University authorities and the trend of current planning is lucidly described by Dr. G. N. Patterson in his paper "The Applied Scientist – A New Member of the Engineering Team" which appeared in the February 1961 issue of the Journal. The assumptions underlying this approach are worthy of examination.

Engineering, as a result of our increasing knowledge, is now a field so vast that most of us are specialists digging ever deeper into a narrowing hole. To be effective it is necessary to group a number of specialists together and coordinate their efforts. This presupposes the existence of men capable of leading the team – a difficult task at the best of times. Without them the specialist is useless. What, then, are we doing to ensure a sufficiency of leaders capable of directing the multiplying specialists?

It is often assumed, as does Dr. Patterson, that the production of many Ph.D.'s will, per se, provide the leaders. The assumption is that deep technical knowledge in a limited area automatically makes an engineering leader. Industry would not agree. Research itself is led in some cases by engineers with a bachelor's degree. The converse, in fact, is true: the greater the number of specialists the more pressing the need for leaders. What, one asks, do we know of the source springs of leadership?

The demise of "handbook" engineers and the empirical approach is predicted on the assumption that engineering work of the future will need superior technical knowledge and will yield only to the application of analytical techniques. Economics alone will dispose of this argument, whatever its absolute merits, but let us be wary lest intellectual snobbery blinds us to the realities of the situation. The part played by engineers whose analytical ability is limited to consulting a handbook accounts for the great mass of day-to-day industrial engineering, and is likely to continue to do so. The glamorous work of scientists in aerospace and electronics, whose contributions form but a small part of the total economy, is usually made manifest and useful by the efforts of handbook engineers.

To assert the analytical and deny the empirical is to deny history. Looking back over the achievements of engineering it is obvious that many contributors to the advance of science and civilization were empiricists. In our world of aviation, as in marine engineering, the pure analyst is a recent arrival, whose very existence is at once dependent upon and supported by the empirical masters. The seed of genius does not reside in a formula.

Of all assumptions, the most important and least supportable is that extensive technical knowledge is the ultimate and only target. We believe that technical education alone will ensure our future. It follows from this premise that the higher the number of Ph.D.'s the better the chances of our survival. The advent of more specialists however, whose contributions are indeed essential, only leads us farther along the path of "committee engineering" as we feel with increasing sharpness the lack of creative leaders, and our products lose the sparkle of originality. We forget that the specialist is seldom the creator of the concept and that he is useful only as a supporter and contributor to the original idea. It is not proved that a Ph.D. is more creative than a B.Sc. or a person lacking a degree at all. The extent of technical knowledge is a secondary factor in determining creativeness and may even inhibit it. The sterility of mathematics is implicit in its own definition.

The assurance of the survival of our engineering society lies in continued creativity. The employment of both scientist and handbook engineer is dependent upon those who conceive ideas and have the force of character to bring the ideas to reality. In laying down the foundations of the education of future engineers it is important to recognize that the technology they will absorb is not an end in itself but a tool to support creative thinking. But what will they be taught of the nature of creative thinking?

Ideas do not occur to a group but in the mind of an individual. Our logic, analyses, technology and objectivity are peripheral activities of conscious reasoning and in themselves create nothing, neither are they capable of creating. The act of creating follows from an idea which arises spontaneously in a realm remote from conscious reason. The idea is then presented to conscious reason for analysis and comparison, a mechanical and sometimes fruitless function. The ability to conceive ideas ensues when the conscious function of analyzing and objectivizing is sublimated, or, in other words, when the mind is allowed to become totally subjective and to identify itself with the medium to be mastered. This process is antescientific, not anti-scientific. What university, in the West, teaches such disciplines?

Research institutes for graduate study are wholly commendable, but a warning should be sounded. In some cases it is customary for such organizations, when not fully utilized, to seek work from industry at low fees. Is it the function of a university to compete unfairly with commercial research, development and testing facilities?

The essence of these arguments is that university authorities would serve society by balancing the teaching of technology with the teaching of individual development. The centrifugal emphasis of objectivity, logics, specialization and group effort are worthless without the power of creative thinking. The centripetal force of subjective, intensely personal growth must be explored and stimulated if the flow of creative thinking is to continue and enlarge.

In teaching the individual the self-discipline required to develop his innate creativity and originality, and hence his personality and dignity, our concern for group compatibility and ethics will dissolve before the warmth of men of character.

Montreal, P.Q. — J. S. Brooks

Dr. Patterson has replied:

My article on "The Applied Scientist – A New Member of the Engineering Team" was written primarily with a view of evoking comments which would be helpful in revisions of our courses at all levels at the Institute of Aerophysics. In addition to Mr. Brooks' letter to the Editor, I have received a number of direct communica-

tions on the subject. I am glad to say that I now possess a substantial file of constructive criticisms and comments. My thanks are due to these people for taking the time to think about it. As would be expected, the truth of the matter appears to lie somewhere between the views of the various participants in this discussion.

Mr. Brooks rightly emphasizes the importance of creativeness and asks what can "be taught of the nature of creative thinking". In attempting a reply I can only call on my own experience such as it is. It is my view that every student comes to us with some potential for creativeness. The process of bringing it to fulness begins with education which must necessarily involve considerable time on the technical knowledge needed to pursue his profession. But there is much evidence that during this period he is developing valuable traits of character, such as perseverance, self-reliance, an ability for logical thinking on subjects in and beyond his course, and a capacity for working harmoniously and effectively with others. In the senior years of his course, the student's program provides opportunities for the development of his capacity for original thinking and his powers of intuition. Finally, if this educational program is followed by appropriate experience and some application of individual initiative, engineering creativeness as we know it today is achieved. It seems to me that it is important to realize that a successful university course does more than impart knowledge – it also induces character.

The leadership of the engineering group, important though it may be, was the subject of one sentence in my article. It refers to "those Ph.D.'s in engineering science who will be our leaders of tomorrow"; but there is nothing in this statement that says that I consider that all leaders in the future will be Ph.D. graduates. Leadership, like gold, is where you find it! On the other hand many of our engineering leaders will come from the ranks of the applied scientists. Mr. Brooks incorrectly assumes that a Ph.D. is a man with only a "deep technical knowledge in a limited area" and presumably lacks the breadth to be a leader. Many of his comments are based on this misconception. What I have said above combined with our own record refutes it. Most of our Ph.D. graduates after a few years' experience have attained supervisory positions of some responsibility. The remainder have preferred to continue contributing effectively in their specialized field. A capacity for leadership is easily detected in a student and we arrange his graduate program, whenever it is possible, to bring out this important quality.

I am afraid that I cannot agree with Mr. Brooks on the subject of the contribution of empiricism to engineering progress. Empiricism is a philosophical theory that the origin of all knowledge can be attributed to experience. On the other hand, observational science combined with theoretical analysis and that remarkable human insight called intuition are the basis of modern engineering. Hydraulics is an example of a largely empirical science. It has been described by Dr. T. von Kármán as the "science of variable constants". Today this subject, distinctly limited in its application, is borrowing heavily from modern fluid mechanics, which from the beginning has resulted from an effective interplay between experiment, theory and intuition. Complete reliance on empiricism alone would reduce engineering to quackery. In my view one of the prime objectives of modern education, research and development is to remove the adamant of pure empiricism from engineering practice.

With regard to the handbook engineer, the writing is already on the wall. Recently in the US automobile industry a whole department of handbook engineers doing routine design was replaced by automation developed by original-thinking engineers using analytical methods. On the contrary, economics will not support the large scale use of handbook engineers. The growth of analytical methods in industry is well attested by the rapid expansion of computing facilities. Not many of us can afford a complex toward mathematics such as that expressed by Mr. Brooks. Educational progress will inevitably continue to stress analytical ability in the student.

Mr. Brooks draws many of his conclusions from his knowledge of industry. I think the reader will be interested in our (UTIA) experience with Canadian industry insofar as it concerns our Ph.D. graduates. The distribution of the latter to date is indicated in the following table according to organization and country:

Nature of Organization	*Number in Canada*	*Number in USA*
Industry	4	11
Government	6	3
Education	7	2
	17	16

The table speaks for itself. It would be interesting to know if this trend is general. In any case it is clear that the program of courses and research at the Institute of Aerophysics for Ph.D. students desiring to work in industry must be slanted largely toward US requirements until the picture changes. Perhaps while we are considering the question of intellectual snobbery we should also take a hard look at the industrial "snubbery" (to coin a new word) evidently practiced in Canada.

Mr. Brooks' point regarding the area of work appropriate for research institutes on the campus is well taken. The policy of UTIA is to stay out of commercial competition. All grants and contracts at the Institute are the result of unsolicited proposals which represent special fields of interest to the staff and are initiated by the staff. Shopping for funds to support this kind of work has become easier in recent years with the setting-up in various agencies of special offices to deal with unsolicited proposals from non-profit organizations. However, it is incumbent on a university research institute to assist government and industry when it possesses the only facilities available.

Finally, my thanks to Mr. Brooks again for a thought-provoking article. I am sure CAI readers will find it as interesting as I have.

Toronto, Ont. — Dr. G. N. Patterson

ANNUAL GENERAL MEETING

Royal York Hotel, Toronto

Thursday, 25th May, 1961

9.00 a.m.

BUSINESS MEETING

President
David Boyd
Vice-President and Deputy General Manager
Rolls-Royce of Canada Limited

Annual Reports of the Council and of the various Committees

10.45 a.m.

GROUND EFFECT VEHICLES

Chairman
Professor B. Etkin
Institute of Aerophysics
University of Toronto

On the Aerodynamics of Curved Jet Sheets
Dr. G. K. Korbacher, Associate Professor
Institute of Aerophysics, University of Toronto

Control Systems for Ground Effect Machines
T. D. Earl
Chief Aerodynamicist, Avro Aircraft Ltd.

10.45 a.m.

ASTRONAUTICS SECTION

Chairman
Dr. P. M. Millman
Head, Upper Atmosphere Research
National Research Council

Annual General Meeting – Business Session

2.30 p.m.

HONOURS AND AWARDS

Chairman
David Boyd
President, Canadian Aeronautical Institute

HONORARY FELLOWS — McCURDY AWARD
F. W. (Casey) BALDWIN AWARD
THE W. RUPERT TURNBULL LECTURE

The Canadian Contribution to the Ground Cushion Story
J. C. M. Frost
Chief Design Engineer VTOL, Avro Aircraft Ltd.

7.00 p.m.

DINNER

Chairman
David Boyd
President, Canadian Aeronautical Institute

The World's Most Dangerous Airline
B. S. Shenstone
Chief Engineer, British European Airways

Friday, 26th May, 1961

9.00 a.m.

AERODYNAMICS OF VTOL/STOL AND HELICOPTERS

Chairman
F. H. Buller
Chief Design Engineer
The De Havilland Aircraft of Canada Limited

Aerodynamic Testing of VTOL/STOL Models Using a Mobile Test Rig
O. E. Michaelsen
Group Leader, Advanced Design
Canadair Ltd.

Running Takeoff Overload Limitations of Some VTOL Aircraft
K. Korsak, Assistant Chief Design Engineer
Piasecki Aircraft Corp.

Test on a Model Fan-in-Wing for VTOL Aircraft
H. S. Fowler, Associate Research Officer
Engine Laboratory, National Research Council

9.00 a.m.

TEST PILOTS SECTION

Chairman
R. J. Baker
Flight Test Engineer-Pilot
Trans-Canada Air Lines

Annual General Meeting – Business Session

11.00 a.m.

PROPULSION SECTION

Chairman
J. J. Eden
Powerplant Engineer
Trans-Canada Air Lines

Annual General Meeting – Business Session

2.00 p.m.

MAN-POWERED FLIGHT

Chairman
Dr. M. G. Whillans
Assistant Chief Scientist (Biosciences)
Defence Research Board

PANEL

Moderator
R. J. Templin
Head, Aerodynamics Section
National Aeronautical Establishment

Members of the Panel
Professor O. Cochkanoff, Nova Scotia Technical College
Dr. J. G. Fletcher, Defence Research Medical Laboratories
R. D. Hiscocks, The De Havilland Aircraft of Canada Ltd.
Professor H. E. T. North, University of Manitoba
B. S. Shenstone, British European Airways

BOOKS

Unsteady Motion of Continuous Media, by K. P. Stanyukovich, Member of the USSR Academy of Sciences (Translated J. G. Adashko and edited by Prof. M. Holt), Pergamon Press, New York, 1960. 745 pages, $15.00.

The above volume is similar in scope but different in content from the well-known text "Supersonic Flow and Shock Waves" by Courant and Friedrichs. As the title indicates there is greater emphasis on nonstationary flows such as self-similar motions, which recently have been successfully applied to hypersonic flows as well (blast wave theory), detonation waves, method of characteristics, spherical rarefaction waves, intense explosions and implosions, underwater blasts and Lagrange's problem of interior ballistics. In addition, there are chapters devoted to nonstationary and steady flows in a gravitational field, some limiting solutions as applied to rarefied media (gas clouds, explosions in space) using the continuum flow equations, and some considerations of relativistic gasdynamics, which have important applications in cosmology and astrophysics.

There is much that can be recommended in the present volume. The author appears to have stressed in great detail those topics in which he has done considerable original work. The result is that many other important aspects of gasdynamics such as high temperature gas flows and hypersonics (for example, imperfect and nonequilibrium gas flows, boundary layers) have not been treated at all or adequately. The present monograph would be more suitable for the student who is already familiar with steady and nonstationary gasdynamics rather than for the beginner.

The book was printed in England using a photo-offset technique in what appears as an attempt to reduce the price to $15.00. However, the final appearance of the text is not conducive to reading or study. The number of illustrations are relatively few, of inadequate size and often oversimplified. A number of errors and misprints were also noted. Nevertheless, the reader who will make a serious effort to become acquainted with the material in this unique book will find that he will have learned a great deal about nonstationary gasdynamics.

I. I. Glass

Axiomatics of Classical Statistical Mechanics, by R. Kurth, Pergamon Press, 1960. 180 pages. Illus. $7.50.

This carefully written text represents a novel departure from the traditional treatment of statistical mechanics. The author has succeeded in treating the subject as a complete deductive system, organizing the discipline along the lines of theorems with elegant proofs. By doing this, the lack of appeal to those with an inclination towards physics is compensated by a gain in mathematical rigour.

The author has attempted to make this book as self-contained as possible by including an auxiliary chapter on abstract set theory, Lebesque integration and theory of Hilbert spaces. These "mathematical tools" are really the background needed for an understanding of the basic principles involved, although the author states in the preface that a knowledge of the elements of calculus and analytic geometry is all that is required.

The author enters his subject with a discussion of mechanical systems characterized by a large number of equations of motion. The principal problem is to find suitable descriptions of average properties of the mechanical system based on the formal concepts of probability theory. One of the main results of this approach is that it leads to a contradiction in regard to Boltzmann's ergodic hypothesis (i.e. in the course of time, the phase average equals the time average of any phase function). A method based on the concept of probability is then presented which clarifies some of the difficulties of the ergodic problem.

To establish the proper relation between quantities defined statistically and the corresponding phenomenological quantities, the equation of continuity and the Eulerian equations of hydrodynamics are deduced. It is interesting to note that these equations agree term by term for points in the interior of a system (fluid); but near the boundary of the system, statistical theory predicts an extra term which is believed to be important in the interpretation of such phenomena as evaporation or surface tension.

By generalizing some of the definitions and earlier concepts, the last chapter treats statistical thermodynamics, thereby establishing the equivalent of the First, Second and Third Law of Thermodynamics as given by the phenomenological theory.

It is rather unfortunate that the application of the theory to physical problems is demonstrated in only a few cases. The examples given help considerably to better understand some of the concepts employed.

There are but few typographical errors; this is rather remarkable for a text book containing so many complicated mathematical symbols.

P. Mandl

The Aeroplane: An Historical Review, by C. H. Gibbs-Smith, H.M. Stationary Office, London, England. 375 pages. Illus. £1 15s.

This is a monumental work, well illustrated with a large number of excellent half tones on 22 plates and 48 line drawings. The index is well detailed making it easy to locate any desired aviation subject.

Part I deals with History in 16 divisions of 146 pages, beginning with Myths like that of Daedalus and Icarus and continuing through the centuries to the invention of the practical aeroplane and the Wright Brothers 1900 to 1905. The narrative then continues on to the First World War and the period following the Second World War. Finally a section is devoted to Modern Aviation and its many problems of high speed and space flight.

What is termed an Interlude of 14 pages is then introduced. This includes interesting quotations of well known persons such as that by Lord Kelvin in 1896; "I have not the smallest molecule of faith in aerial navigation other than ballooning." And by Orville Wright in 1917; "When my brother and I built and flew the first man carrying flying machine, we thought that we were introducing into the world an invention which would make further wars practically impossible."

Part II, The Commentary, deals with 23 various origins and other important events in 113 pages. Claims to First Powered Flights and other controversies are also gone into. A Table of "First Efforts to Fly a Powered Aeroplane and Principal Early Flights (1874-1909)" is included in this section.

This is a most interesting book so much so that one is likely to be lulled into not observing omissions. It is incumbent on Canadians to uphold what Frank Ellis has called "Canada's Flying Heritage" about which he wrote so ably. Under no circumstances should mistakes or omissions in Canadian aviation history go unanswered.

I feel that the work of the Aerial Experiment Association of Canada has not been given adequate coverage, for, after all, they worked out in the open for all to see and were the first in America to use ailerons and the steerable undercarriage. In this they received no assistance from the Wrights. When the Association was dissolved on March 31st, 1909, Baldwin and McCurdy carried on building aircraft. Commercially this venture was unsuccessful and Baldwin gave up flying and later on developed hydrofoil boats. John McCurdy, however continued flying and became a legendary figure in America. He was the most skillful of the AEA pilots. The author records little of this, preferring apparently to refer to Curtiss whenever possible, as I shall point out.

On Page 19 there is a statement that lateral stability and control posed a vital problem to Langley, Wright, Curtiss and the whole European school. Why mention only Curtiss and not the AEA? It was Dr. Bell who suggested the wing tip ailerons first used on Baldwin's White Wing in May 1908. The AEA did not invent the aileron but there is no doubt that they were among the first to produce practical ailerons acting to increase lift on one side and decrease it on the other side of the span.

The statements on Page 61 give a very inadequate picture of the work of the AEA. It is described as "an organization founded by Dr. Bell (of telephone fame), Glen Curtiss and others". Surely it should have been definitely stated that the members were F. W. (Casey) Baldwin, J. D. McCurdy, Lieutenant Selfridge and Glen Curtiss.

On Page 182, there is the statement that Curtiss invented the method of operating ailerons on his aircraft. This is not correct. The method was first used by Baldwin on the White Wing. The patent was registered in the names of all members of the Association.

The derivation of the word "aileron" is given on Page 184 as a French word designating the extremity of a bird's wing tip. I find this to be reasonably correct. The Life Science Division of the Royal Ontario Museum looked this word up and found that the aileron is the pinion or distal terminal end of a bird's wing. Mr. Gibbs-Smith states that the word only came into use in England in 1909. Pages 179 to 184 give a very illuminating history of the development of wing warping and ailerons for lateral control. Many seem to have tried some form of aileron and then to have come back to wing warping and vice versa. The Bleriot machines flown by Count De Lessops in the second Canadian Air Meet at Tretheway Field near Toronto, Canada, from July 9th to 13th, 1910, used warping. On all this background I repeat part of the story told by John McCurdy about his meeting Henri Farman at an Air Meet outside New York City in September 1908.

"Farman crawled into his machine and ran down the race track, took off and flew at an elevation of five feet for about one hundred yards and landed. His mechanic turned the machine around and he flew back. I asked him why he did not go up higher and make circles around the track. He exclaimed that it was impossible because the machine would tip over." McCurdy then explained what they had done with a little wing and Farman exclaimed, "Ah, aileron!" This has been assumed by many to be the first use of the word "aileron", which is not correct. It was probably the first use of the word in North America. But the statement on Page 183, from a letter dated June 20th, 1908, from Wilbur to Orville Wright about Farman having used auxiliary surfaces on the wing tips of some form, would tend to show that Farman recognized McCurdy's explanation as something he had already seen and used when he is said to have exclaimed "Ah, aileron!" A question which must occur to everyone on reading the statements about Farman on Page 183 is why did he appear in September 1908 at the New York Meet with no lateral control on his machine?

There is no mention made of W. R. Turnbull of Rothesay, N.B., who in 1902 constructed one of the first wind tunnels in North America, investigating airfoil sections, propellers and hydrofoil supported boats. His publications on aerodynamics were widely recognized. The Royal Aeronautical Society elected him a Fellow and awarded him their Bronze Medal. He suggested to Dr. Bell the wing airfoil section used on the Curtiss June Bug. His variable pitch propeller was a great success and was sold in 1929 to the Curtiss Wright Corp. in the USA and to the Bristol Aeroplane Co. in England.

There is also no mention of McCurdy's flight from Key West, Florida, to Havana, Cuba, in January 1911. This was certainly the longest flight over ocean water at that time. The fact that he landed in the shark infested water outside Havana harbour, and not on the flying field, might be taken by some makers of rules as a reason why the event should not be recorded. And yet on Page 69, failures to cross the English Channel completely are recorded.

It is hoped that some form of Errata can be included in unsold copies and offered to those who have purchased this excellent book.

T. R. Loudon

NOTICES TO AIRCRAFT MAINTENANCE ENGINEERS

Issued by the Department of Transport

The Department of Transport, Air Services, Civil Aviation Branch, has promulgated the following Notices to Aircraft Maintenance Engineers:

No. 1/60

Issue 1: November 24, 1960

1. As a method of distributing information "Notices to Aircraft Maintenance Engineers" are being introduced and will be published at irregular intervals as required to draw attention to:

(a) amendments to the Engineering and Inspection Manual (amendments will not be reprinted, only a notice that it is available);

(b) issue of Federal Aviation Agency Airworthiness Directives (these Directives will not be reprinted, the subject and title only will be given);

(c) items regarding inspection, maintenance, certification, etc, which may appear necessary from time to time;

(d) issue of Air Navigation Orders or amendments thereto (the Orders will not be reprinted, the subject and title only will be given);

(e) issue of Canadian Airworthiness Directives which in this case will be repeated pending their incorporation in the Engineering and Inspection Manual; and

(f) any other matters of interest.

2. Notices will be sent free to Aircraft Maintenance Engineers, approved operators and manufacturers. Prompt notification of change of address will ensure continued receipt of these Notices.

No. 2/60

Issue 1: December 15, 1960

As part of Amendment No. 12 to the Engineering and Inspection Manual in 1959, the previously used system of Release Notes was changed somewhat to reduce the number of A.I. forms and to provide an inspection certificate in tag form (A.I.-99) which could be attached to a component or part and so provide definite identification and proof of the airworthiness status of the part.

The sample tag was prepared to show the minimum information required on it and should, without any alterations, be suitable for the majority of operators. In the case of operators who may wish to have additional information on the tags for their own records, this change is permissible.

Part I, Chapter III, of the Engineering and Inspection Manual requires the use of Tag A.I.-99 and this notice is issued to explain further. Tags are required on all components, instruments, accessories or parts which carry a serial number, or are of such a size and design that they cannot be classed as general aircraft hardware (nuts, bolts, bushings, pins, etc). If, for instance, a shipment of parts is received from a supplier in the U.S.A. with a certified invoice, the parts should be tagged before leaving quarantine stores. This then gives a definite identification of the item, its source and proof of airworthiness.

Canadian suppliers making mixed or bulk shipments of, for instance, 'O' rings, studs, spark plugs, piston rings, etc, would not be required to attach A.I.-99 tags to each, but could certify the shipment by means of the general certification (para. 3.1.16) on the invoice. However, if a cylinder assembly or a magneto is included in the shipment, this item must have a tag attached to it.

It is not necessary to replace tags on components or parts received and placed in bonded stores for resale or use by purchaser. The part may be delivered with the original tag affixed. In the case of an assembly received covered by one tag, such as a wheel assembly, and it is later decided to deliver the tire and tube to one customer and the hub to another, it would then be necessary to originate tags for each item, making reference to the original tag on the complete assembly.

No. 1/61

Issue 1: February 8, 1961

New Air Regulations have been promulgated in the *Canada Gazette*, Part II, Vol. 95, No. 1, dated January 11, 1961, under authority of PC 1960-1775.

No. 2/61

Issue 1: February 8, 1961

The F.A.A. have promulgated the following Airworthiness Directives:

#61-1-1 – applying to Cessna 150 Models (superseding A.D. 60-1-2).
#61-1-3 – applying to Sikorsky S-58 Models.
#61-2-1 – applying to aircraft fitted with certain Graviner automatic fire extinguisher cartridges.

No. 3/61

Issue 1: February 24, 1961

Amendment List No. 13, extensively revising the Engineering and Inspection Manual, is now available from the Queen's Printer. It also contains copies of Airworthiness Directives 59-3 through 60-8. Those who have not already subscribed to the amendment service should obtain copies of this amendment.

Owing to an error in printing, the replacement page containing Part I, Chapter III, Paragraphs 3.1.13 and 3.1.14 also contains the last five lines of Paragraph 3.1.8(b); and the replacement page containing Paragraphs 3.1.17 and 3.1.18 contains the last nine lines of Paragraph 3.1.14(c). Consequently, each of these pages should be cut and the appropriate paragraphs placed together.

No. 4/61

Issue 1: March 1, 1961

The F.A.A. have promulgated the following Airworthiness Directives:

#61-3-1 – applying to Bell Model 47J-2 Helicopters.
#61-3-2 – applying to Continental E165, E185, E225 and O-470 series engines.
#61-3-3 – applying to certain Hartzell propellers.
#61-3-5 – applying to Luscombe Model 8 series aircraft.

No. 5/61
Issue 1: March 6, 1961

The following Airworthiness Directives have been issued subsequent to No. 60-8, the last one contained in Amendment List No. 13 of the Engineering and Inspection Manual:

61-1
PBY-5A
In operating any Consolidated-Vultee Model PBY-5A aircraft in air carrier passenger service (A.T.B. Class 6 and Class 7 air carrier services excepted) after 31st December, 1961, the following shall be complied with:

At or near the firewalls, emergency shut-off valves shall be incorporated in all lines carrying inflammable fluid into the engine compartments. If these valves are located forward of the firewall, they and any other system components between the valves and the firewall must be fireproof or adequately protected by fireproof wrapping or stainless steel shrouds. The operating means shall be located convenient to the pilot and co-pilot or to the flight engineer and shall be properly marked.

61-2
Cessna Model 150 Aircraft
Intentional spinning of Cessna 150 aircraft is prohibited. Therefore the placards required by Note (2)(a)(2) of FAA Type Certificate Data Sheet No. 3A19 must be modified as follows:

delete "Spins Use slow decelerations"
add "Intentional Spins Prohibited"

Any pilots' notes, manufacturers' manuals, etc, are to be amended accordingly. A log certification that this directive has been complied with must be made. Compliance is required before next flight, except for Cessna Model 150 aircraft Serial Nos. 17719, 59019, 59049, 59057, 59061, 59062, 59067 and on, and aircraft modified in accordance with Cessna Service Letter 150-21.
This supersedes Airworthiness Directive 60-4.

61-3
DHC-2 Beaver Aircraft
Replacement of retractable main ski axle pins and bushings Part Nos. C3-US-155-5 and C3-US-149-3 by Part Nos. C3-US-286-3 and C3-US-287-3 respectively, as described in de Havilland Engineering Bulletin Series "B" No. 24, applicable to all DHC-2 Beaver aircraft equipped with combination wheel-skis, is mandatory. Compliance is required at the next ski installation, or not later than 15th February, 1961, or may be postponed until the aircraft is at a base where adequate facilities for doing the work are available provided an immediate inspection is made for excessive play or seizure at the axle pin and that the area is adequately lubricated and provided this inspection and lubrication is repeated at one hundred hour intervals until replacement of part is accomplished.
This supersedes Airworthiness Directive 60-6.

61-4
DHC-3 Otter Aircraft
Replacement of retractable main ski axle pins and bushings Part Nos. C3-US-155-5 and C3-US-149-3 by Part Nos. C3-US-286-3 and C3-US-287-3 respectively as described in de Havilland Engineering Bulletin Series "O" No. 70, applicable to all DHC-3 Otter aircraft equipped with combination wheel-skis, is mandatory. Compliance is required at the next ski installation, or not later than 15th February, 1961, or may be postponed until the aircraft is at a base where adequate facilities for doing the work are available provided an immediate inspection is made for excessive play or seizure at the axle pin and that the area is adequately lubricated and provided this inspection and lubrication is repeated at one hundred hour intervals until replacement of part is accomplished.
This supersedes Airworthiness Directive 60-8.

These Directives will be included in Amendment List No. 14 to the Engineering and Inspection Manual.

No. 6/61
Issue 1: March 15, 1961

The F.A.A. have promulgated the following Airworthiness Directives:

#60-12-4 – applying to Fairchild F-27 Series.
Revision
#61-4-1 – applying to Brantly B-2 Helicopters.
#61-4-2 – applying to Douglas DC-6 Models.
#61-4-3 – applying to Walter Kidde Company Chemical Drier Housings used on Fairchild F-27 Series.
#61-4-4 – applying to Lockheed PV-1 and B-34 Aircraft.

No. 7/61
Issue 1: March 15, 1961

In Amendment List No. 13 a misprint occurred in the second line of sub-paragraph 1.7.3(b), Chapter I, Part II, of the Engineering and Inspection Manual. The word "commercial" in this line should be changed to "scheduled" so that the sub-paragraph should read:

"(b) private aircraft and commercial aircraft of less than 12,500 lb gross weight, except when used on scheduled air service, etc., etc."

No. 8/61
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Inquiries have been received regarding Department of Transport approval of "Vitalyte" and its use as an electrolyte replacement in lead acid type aircraft storage batteries.

The Department has not approved "Vitalyte" as an electrolyte replacement, since it does not issue blanket engineering approval for such items as battery additives or replacement electrolyte. It is permissible, however, to use a replacement electrolyte provided the electrolyte replacement used is endorsed by the battery manufacturer involved, or alternatively the same has been substantiated by a performance test conducted by the user to demonstrate that the replacement will not reduce performance. An acceptable performance test would be to compare the capacity of the battery at the 5-minute, 1-hour, and 5-hour discharge rates (using the original electrolyte) with the capacity of the same battery at the same discharge rates using the replacement electrolyte. This test should be conducted at both 0° and 75°F. and the electrolyte replacement should be used according to the manufacturer's instructions. If a reduction in performance is noted, the replacement electrolyte is not acceptable.

It should be noted that the onus is on the user of an electrolyte replacement to either provide the battery manufacturer's endorsement or conduct a performance test on the battery in which he desires to use the replacement electrolyte.

The Applied Research Laboratories test report on "Vitalyte" (copy of which we have on file) covers the test on a new battery only, and indicated the average performance of the test battery was not appreciably reduced. The test report, however, did not indicate the effect of "Vitalyte" on the life of the battery or its effectiveness as a substitute electrolyte in old batteries.

C.A.I. LOG

SECRETARY'S LETTER

THE JOURNAL

In this issue, the Canadian Aeronautical Journal assumes a new form. Of necessity, certain economy measures have been thrust upon us and we hope that the loss of some of the features of past issues will not seriously affect the Journal's usefulness as a technical publication or as a bond between the members of the Institute. We have had to discard the frontispiece and the Editorial, and we have dispensed with separate sections of the C.A.I. Log devoted to Branches, Sections, Members and Institute Meetings; in their place this Letter will try to report on the highlights and to summarize the Institute's activities month by month. Items which can be classed as notices will appear in the C.A.I. Log under Announcements. The technical content, the papers, forum and book reviews, will remain unchanged.

THE BRANCHES

Reports received from Branch Secretaries since the last issue of the Journal cover the January meetings held in Quebec and Winnipeg and the February meetings in Montreal, Toronto, Ottawa, Halifax-Dartmouth and Calgary. Taking them in this order:

Quebec — 26th January

Dr. J. J. Green, Chief Superintendent of CARDE and, of course, one of the Canadian representatives of the International Council of the Aeronautical Sciences, gave a report on the Second Congress held in Zurich in September and summarized in some detail about half a dozen of the papers presented. It appears from Professor Nicholl's report that this was similar to Dr. Green's talk to the Ottawa Branch which was recorded in the February issue and, if so, the Quebec Branch had a very worthwhile evening.

Winnipeg — 31st January

Mr. E. Rice of the Aerojet-General Corporation spoke on "The Economic and Safety Aspects of Stand-by Rocket Power for Aircraft". The meeting was held for the first time in the Officers' Mess, RCAF Station Winnipeg. From Mr. Baker's account it appears that the speaker made a fairly convincing case for stand-by rocket equipment as a relatively cheap form of insurance against aborted takeoffs and reduced payload takeoffs under unfavourable conditions.

Montreal — 15th February

This meeting was run by the Montreal Student Section. Mr. I. Wygnanski of McGill University delivered a paper based on his winning submission to the Essay Competition and entitled "An Approximate Method of Predicting Velocity Distribution in Rectangular Channels". He was presented with the Branch Student Award. The meeting also included a demonstration of an electrically powered ground effect machine, constructed by the Fifth Year Mechanical Engineering students at McGill, and a talk on this machine by Mr. C. T. Ogryzlo, one of those who had worked on it. Several members had a ride on it, including the President, who has told me that he had been most impressed. There were 62 members and guests present.

Toronto — 15th February

This was the Branch Students' Thesis Night. This usually very well-attended event had the misfortune to occur just after a heavy snowfall, which reduced attendance this year to 49. Three University students, Mr. W. H. Mak, Mr. I. Banks and Mr. R. C. Radford presented papers entitled "Prandtl-Meyer Expansion of Reacting Gases in Equilibrium and Frozen Flows", "The

Montreal: (l to r) Mr. R. J. Conrath, Student Activities Committee; Mr. R. B. Thomas, Chairman, Student Section; Mr. I. Wygnanski, Essay Competition Winner; and G/C C. W. Crossland, Student Awards Committee, examining the McGill GEM.

Halifax-Dartmouth: (l to r) Mr. F. T. Dryden, Publicity Committee; LCDR G. M. Cummings, Chairman; Mr. I. MacDonald, Speaker; and Prof. O. Cochkanoff, Vice-Chairman.

Problem of Turbine Blade Cooling in Gas Turbines" and "An Application of Photo-elasticity to Experimental Stress Analysis", respectively. A panel of three judges selected Mr. Radford's paper as the best of these theses and the Branch Student Award was accordingly presented to him by Dr. G. N. Patterson. Another Award was presented to Mr. N. M. Smith of the Ryerson Institute of Technology for his thesis entitled "Evolution of Jet Propulsion" – this was not presented orally; and a third prize winner was Mr. M. Wellman of Central Technical Institute, who had been selected as the outstanding pupil in the aeronautical course.

Ottawa — 15th February

W/C K. R. Greenaway, Officer Commanding, Central Navigation School, Winnipeg, presented a paper entitled "A Navigation System for a Mach 2 Transport". The disappointing attendance of 57 included a number of students from the RMC. I hope that in due course we shall be able to publish this paper in the Journal – W/C Greenaway promised me a copy of his manuscript – for, as might have been expected from such an authority, it was very good and provoked an unusual amount of discussion.

Halifax-Dartmouth — 15th February

Mr. I. MacDonald of TCA, Project Engineer on the Vanguard, gave a talk entitled "TCA's Vanguard" and showed a film depicting stages in the production and testing of this aircraft. The meeting was held in the RCAF Officers' Mess, Anderson Square; this was a change from the cinema of the CPO's Mess at RCN Air Station, Shearwater, where the Branch usually meets. The attendance was 39.

Calgary — 16th February

Mr. M. Hamilton, Station Manager of West Coast Air Lines gave an account of short haul, fast turn-around operations in a talk entitled "America's Local Service Air Lines". There were 26 present and, as usual at Calgary, this was a dinner meeting.

THE SECTIONS

In addition to a fairly active month at the Branches, there have been meetings of the Montreal Groups of the Propulsion and Astronautics Sections.

Montreal Propulsion Group — 15th February

On the 15th February a meeting was held at Canadair Ltd. to discuss the subject "Keep Oil in Engine – Can it be Done?" Mr. J. J. Eden, the Chairman of the Section, presided and 52 members and guests were present. Three speakers, Mr. A. B. Newland of Canadian Pratt & Whitney, Mr. J. D. McArtan of TCA and Mr. F. G. Massey-Shaw of Canadair presented short papers on various aspects of the problem of oil leakage and they were followed by a vigorous discussion which, to judge from W/C Londeau's report, went on merrily until 10.45 pm.

This was the first such meeting by the Propulsion Section and it promises well for the future.

Montreal Astronautics Group — 23rd February

Dr. H. J. Luckert, Chairman of the Montreal Group of the Astronautics Section, tells me that the Group held a successful meeting on the 23rd February, addressed by Dr. M. B. T. George of AVCO, on the subject of "Lunar and Cis-Lunar Spacecraft", but at present I have no details to report.

MID-SEASON MEETING

The Mid-season Meeting was held in the Marlborough Hotel, Winnipeg, on the 27th and 28th February and, like the Toronto Students' Thesis Night mentioned above, it too suffered from the weather. Freezing rain descended on Montreal and Ottawa and, particularly in Montreal, a good many members who were planning to come had to stay at home to make the best of candle light and cooking on the sitting room hearth. Consequently the total registration was a disappointing 89; so good a programme, as set out in the last issue of the Journal, deserved a better fate.

However the Dinner on the evening of the 27th February was most successful. The distinguished guests included Mr. B. Strickland, MLA, representing the Premier of Manitoba, A/V/M H. M. Carscallen, AOC Training Command, RCAF, and Mr. R. L. Bolduc, Regional Controller of Civil Aviation, DOT. The Guest of Honour

The President speaking at the Dinner of the Mid-season Meeting: (l to r) A/C W. P. Gouin, Vice-President; Mr. G. W. G. McConachie, Principal Speaker; the President; Mr. B. Strickland MLA; and, beyond the lectern, Mr. R. L. Bolduc, Dept. of Transport.

and Principal Speaker was Mr. G. W. G. McConachie, President of Canadian Pacific Air Lines, whose address appears on pages 147 to 151. Mr. Strickland spoke briefly and amusingly before Mr. McConachie. The dinner attendance was 164.

Reverting to the technical sessions, as I have said the programme was excellent and I hope that we shall be able to publish some of the papers in full later on, so that they will be made available to a wider audience than they were vouchsafed at the Meeting itself. But the rooms we had chosen were too big, emphasizing the poor attendance.

Even so it was good to see quite reasonable delegations from the Vancouver and Calgary Branches and we all greatly enjoyed the Winnipeg hospitality.

LIST OF MEMBERS

The usual "buff cards", setting out the particulars we require for the List of Members, are being distributed with members' dues bills. It is most important that these should be completed and returned by the 15th May; unless we have them, we can list only the last mailing addresses known to us, which is rather unsatisfactory. Please let us have them back promptly.

ANNOUNCEMENTS

SUSTAINING MEMBERS

The following Sustaining Members joined the Institute during the year 1960-61:

Bourne & Weir Limited
BP Canada Limited
Irvin Air Chute Limited
Timmins Aviation Limited
Fort Gary Tire & Auto Supplies

ADMISSIONS

The following is a list of admissions and advancement in grade of members during the month of February 1961:

Associate Fellow

Dr. G. V. Bull (*from Member*)
W. B. McCarter

Member

R. Adams
C. J. Barker
CPO G. E. Blackwell (*from Technical Member*)
F/L E. R. Chappell
J. N. Clark (*from Technical Member*)
K. C. Falconer
J. E. Fortune
F/L H. R. Grainger
WO H. D. Haigh
R. J. Losch
H. G. MacIntosh
D. C. Parke
L. A. Pragnell
J. P. Rickard (*from Technical Member*)
R. L. Samuels
G. B. Spindler
L. V. Ursel (*from Technical Member*)

Technical Member

F/O A. S. Armstrong (*from Student*)
J. H. S. Conn
F. W. Diamond
WO T. C. Forrester
F/O E. Lukan (*from Student*)
D. S. Miller
R. W. Murray
Cpl H. A. North
E. Palka
FS J. A. Sagar

Student

F. D. Adkins
I. Banks
G. F. Beals
W. H. Bell
M. K. Campbell
R. M. Chown
R. J. Ellery
F/C J. B. Feir
J. H. D. Gladstone
T. Graham
E. Haasdyk
H. A. MacDonald
P. March
N. A. Martin
W. A. McKenzie
C. T. Ogryzlo
D. W. Owen
A. A. Reimer
G. H. Simms
A. V. Sloan
Y. W. Tsang
D. W. Washburn
R. E. Wayland

Associate

R. A. Munro

OBITUARY

W. D. Hunter, F.C.A.I.

Walter Douglas Hunter, 66, a Fellow of the Canadian Aeronautical Institute, died suddenly on February 2nd, 1961, in Toronto. He had been at work in his office and in good spirits only two days previously, and his death came as a great shock to all those who were close to him in his life and work.

Born in England in 1894, Doug Hunter began his career in aviation as an apprentice with Graham White in 1913. From that time onwards he was continuously engaged in the design and engineering aspects of aviation. In 1919 he was with Kennedy Aviation on the design of trans-Atlantic aircraft. From 1920 to 1922 he was with Vickers as designer on the airship R-80 and their large flying boats. For the next three years he was with Fairey Aircraft Co. Doug often reminisced about the early days of flying and in the last few years had collected many interesting notes, pictures and drawings of the early aircraft which he had known at first hand in England.

In 1925 Doug began his long association with Captain (now Sir Geoffrey) de Havilland and the de Havilland Aircraft Co. For 16 years he was at Hatfield where he participated in the creation of many famous DH types – the DH-66, DH-60 Moth, the DH-61, the DH-65 Hound, the Tiger Moth, Puss Moth, Dragon, Dragonfly, Rapide, DH-86 and the DH-98 Mosquito. During those days he became a part of what is known throughout the industry as the "DH philosophy" – an approach to aviation which has shunned the complicated in favour of the commonsense and straightforward.

During World War II when England wanted Mosquito aircraft built in Canada, Doug was seconded to the de Havilland Aircraft of Canada at Downsview. He arrived in Canada in 1941 and was put in charge of Engineering at the DH Canada factory. What he then thought was to be only a war-time post, expanded and turned out to be the major portion of his DH career. He was proud of the fact that he had served de Havilland longer in Canada than he had in England.

In 1946 he was appointed to the Board of Directors of DH Canada in charge of Engineering. Under his direction DH Canada brought forth the Fox Moth (DH-83-C), the Chipmunk (DHC-1), the Beaver (DHC-2), the Otter (DHC-3) and recently the Caribou (DHC-4). These successful aircraft remain as a tribute to the skill which Doug displayed in steering a steady engineering course through many difficult situations. He was

basically a shy man of quiet dignity, and these characteristics earned him much respect amongst his colleagues. In his thoughtful way he often took great pains to resolve problems to the benefit of both his Company and his people. He lived for his work and enjoyed maintaining an atmosphere of cheerful challenge amongst his associates.

Doug Hunter's faith in Canadian aviation was further demonstrated by the help and support he gave the CAI during its formative years. He was a member of the Council in 1955-56 and 1956-57. Never a man to seek the limelight, he worked quietly behind the scenes to help Canadian aeronautical engineers and technologists have an organization which would increase their status in the eyes of the world.

The Canadian aviation industry has lost one of its most ardent devotees and engineering leaders. We have lost a lifetime of experience which spanned the lifetime of the aeroplane. We have also lost a great friend. We believe he knew that he had infused his associates with some of the DH philosophy. And this pleased him.

R. B. McINTYRE

BRITISH INCOME TAX

The Canadian Aeronautical Institute has been approved for the purposes of Section 16 of the British Finance Act 1958. Members who qualify for relief under that Section will be allowed to deduct the whole of their dues from their emoluments assessable to income tax under Schedule E. A member who is entitled to this relief should apply to his tax office for Form P 358 on which to make a claim for the relief due to him.

APPOINTMENT NOTICES

The facilities of the Journal are offered free of charge to individual members of the Institute seeking new positions and to Sustaining Member companies wishing to give notice of positions vacant. Notices will be published for two consecutive months and will thereafter be discontinued, unless their reinstatement is specifically requested. A Box No., to which enquiries may be addressed (c/o The Secretary), will be assigned to each notice submitted by an individual.

The Institute reserves the right to decline any notice considered unsuitable for this service or temporarily to withhold pulication if circumstances so demand.

POSITION WANTED

Box 111 **Technician:** Aircraft Maintenance Technology graduate from the Southern Alberta Institute of Technology and Art desires position in industry. Will appreciate all offers. Excellent references available.

SUMMER EMPLOYMENT

Box 112 **Student:** Second year engineering student at Ecole Polytechnique de Montréal seeks position for the period 15th May to 15th September. Bilingual with technical drawing and surveying experience.

Box 113 **Student:** First year student in Aircraft Maintenance Technology at the Southern Alberta Institute of Technology and Art seeks position, in western Canada, for the period May 15th to September 1st.

Box 114 **Student:** First year student in Aircraft Maintenance Technology at the Southern Alberta Institute of Technology and Art seeks position, preferably in Ontario, for the period May 15th to September 15th.

Box 115 **Student:** Student having completed his first year in the Aircraft Maintenance Technology course at the Southern Alberta Institute of Technology and Art seeks position in the Edmonton or Calgary areas.

Box 116 **Student:** Second year postgraduate student in Mechanical Engineering at Nova Scotia Technical College seeks position from the 1st May in the propulsion field, with an outlook for permanent employment after graduation. No geographical preference. Experienced in supervision.

Box 117 **Student:** First year student in Aircraft Maintenance Technology at the Southern Alberta Institute of Technology and Art seeks position, preferably in western Canada, for the period March 20th to September 1st. Hard worker and willing to learn.

Box 118 **Student:** Second year student in Aeronautical Engineering at the Southern Alberta Institute of Technology and Art seeks position, preferably in western Canada, for the period May 15th to September 15th. Experience includes two years mechanical, structural and topographical drafting.

Box 119 **Student:** First year student at the Southern Alberta Institute of Technology and Art seeks position anywhere in western Canada. Experience includes a private pilot's licence and two months service in the RCAF Reserve Tradesmen Training Plan towards an Airframe Technician Group I rating.

Box 120 **Student:** Fourth year Mechanical Sciences student at McGill University seeks position in plant engineering. Experience includes four years workshop and drafting prior to entering university.

Box 121 **Student:** Second year student in Aeronautical Technology at the Ryerson Institute of Technology seeks position for the period May 1st to September 15th.

Box 122 **Professional Engineer:** M.C.A.I. presently undergoing postgraduate studies at university seeks position for the period June 10th to October 5th. Experienced in research, development, design and evaluation in aircraft and missile structures, propulsion, control instrumentation, hydraulics, heat transfer, jig and tool design etc.